Is the fractal approach useful to create future cities and regions?

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PART I: INTRODUCTION TO FRACTALS IN THE BUILT ENVIRONMENT

Latin verb frangere = to break
Adjective fractus = irregular, fragmented

The term fractal is used to describe objects that demonstrate a repeating pattern across scales = self-similarity

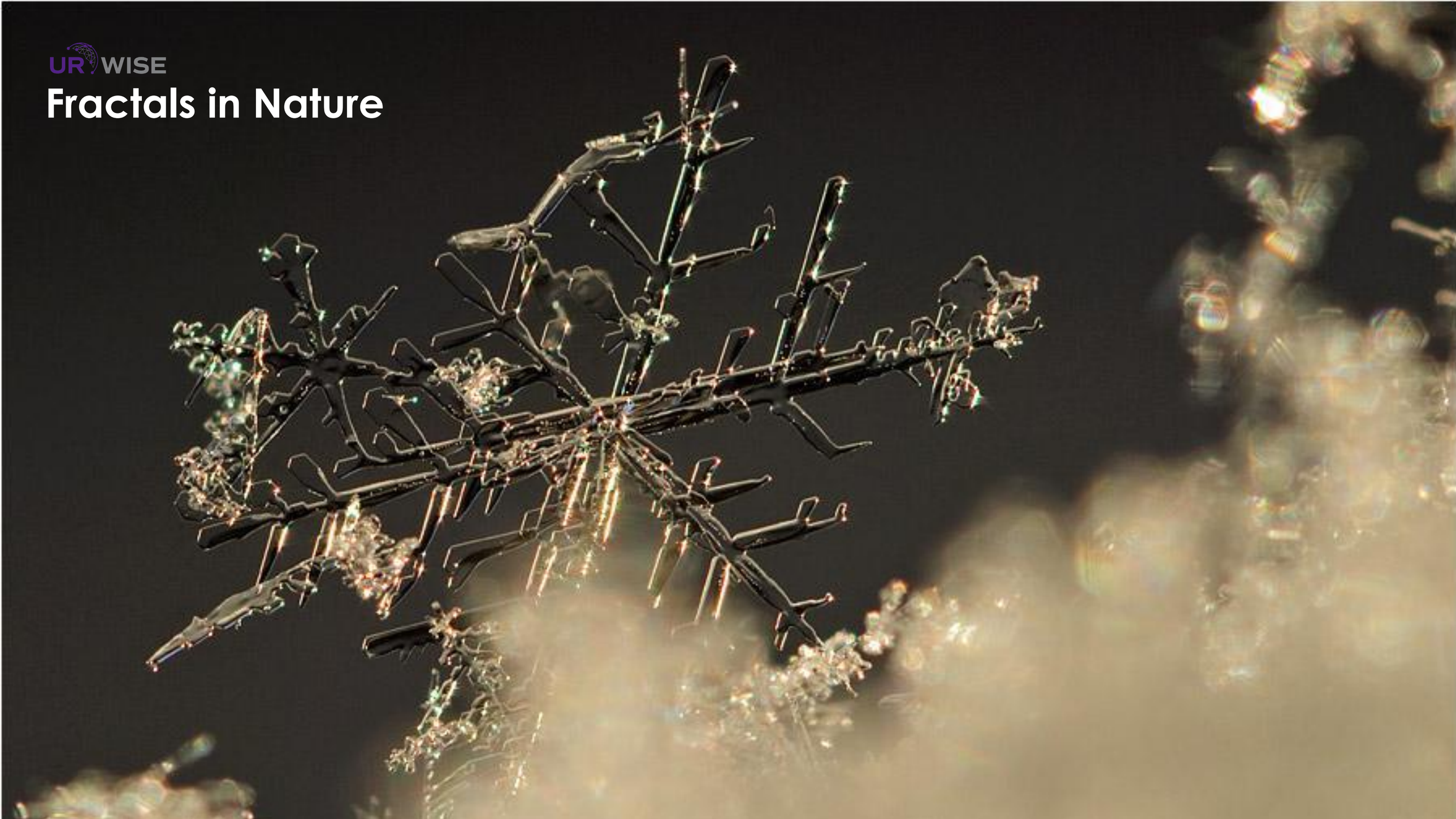
Mandelbrot, 1977

URWISE

Fractals in Nature

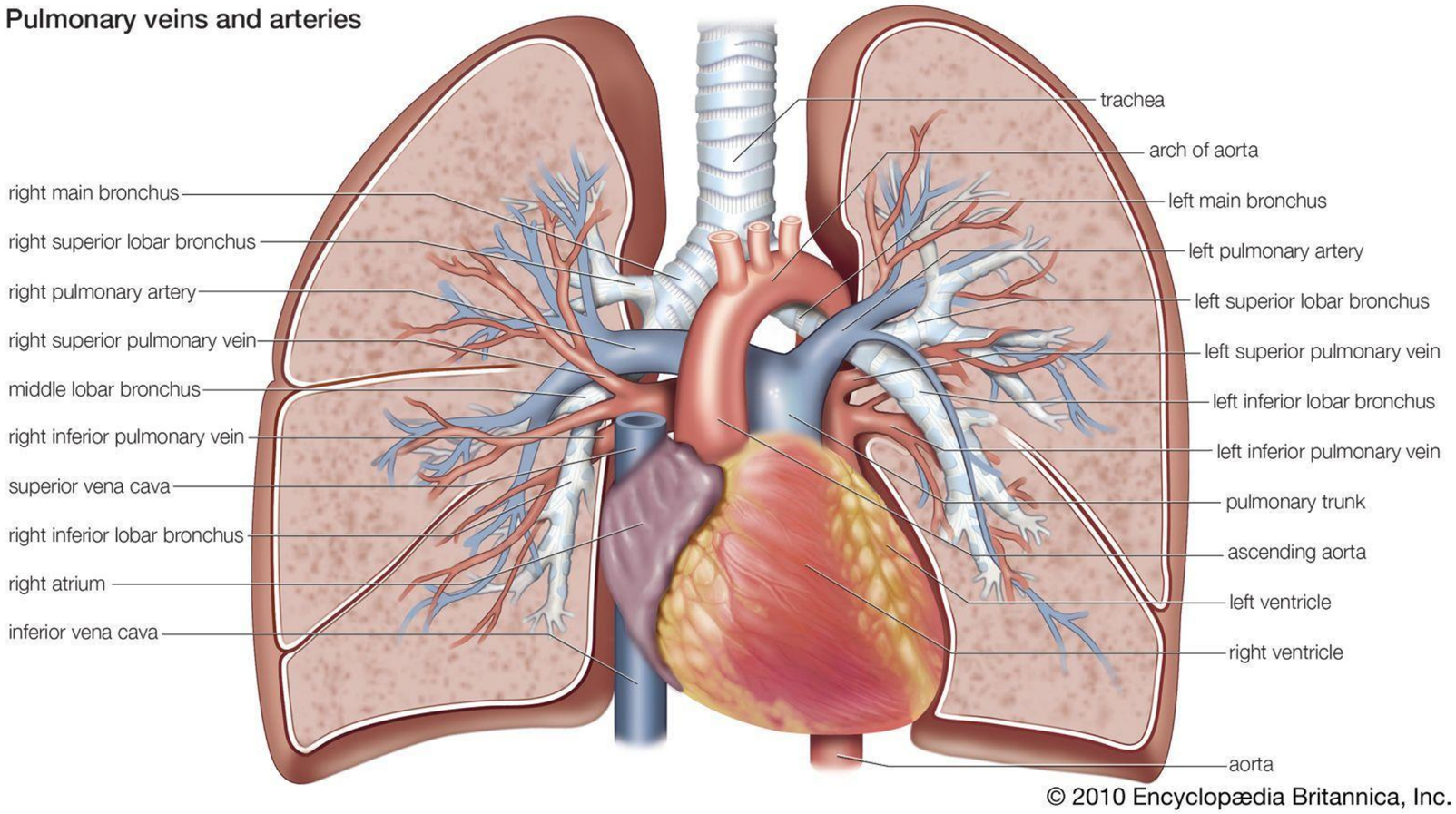
URWISE

Fractals in Nature

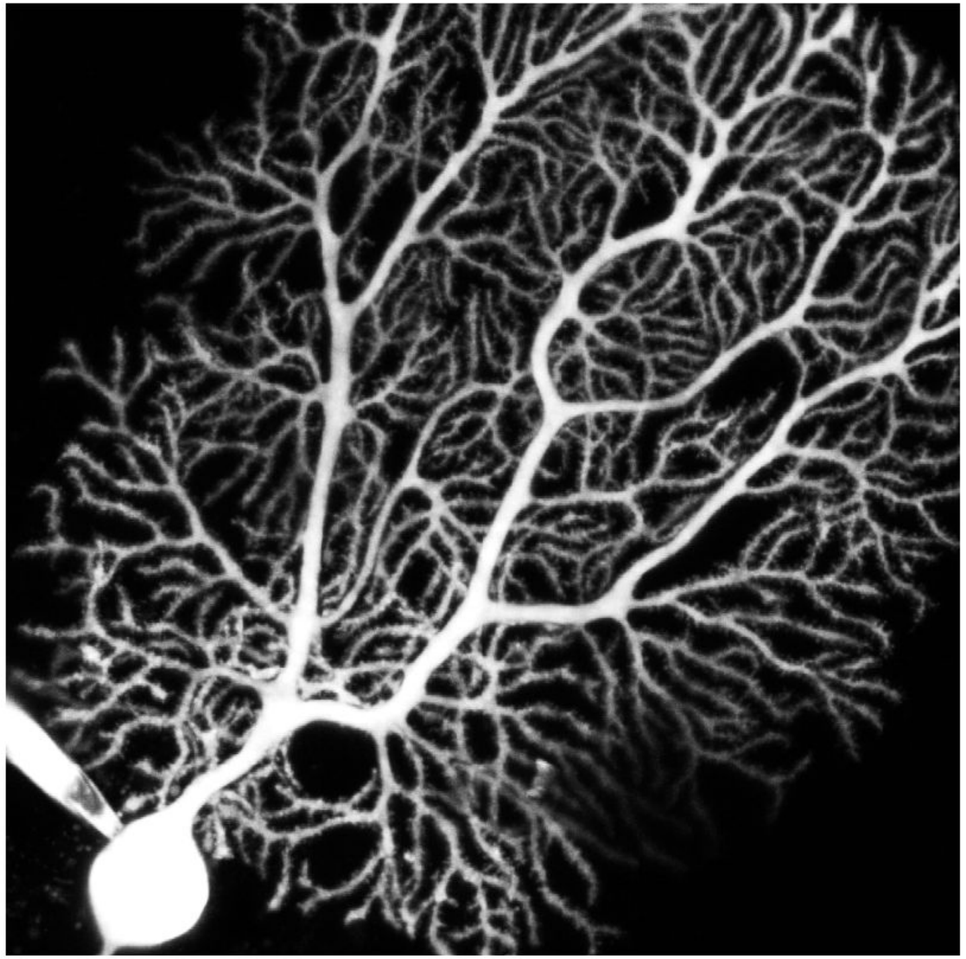


URWISE

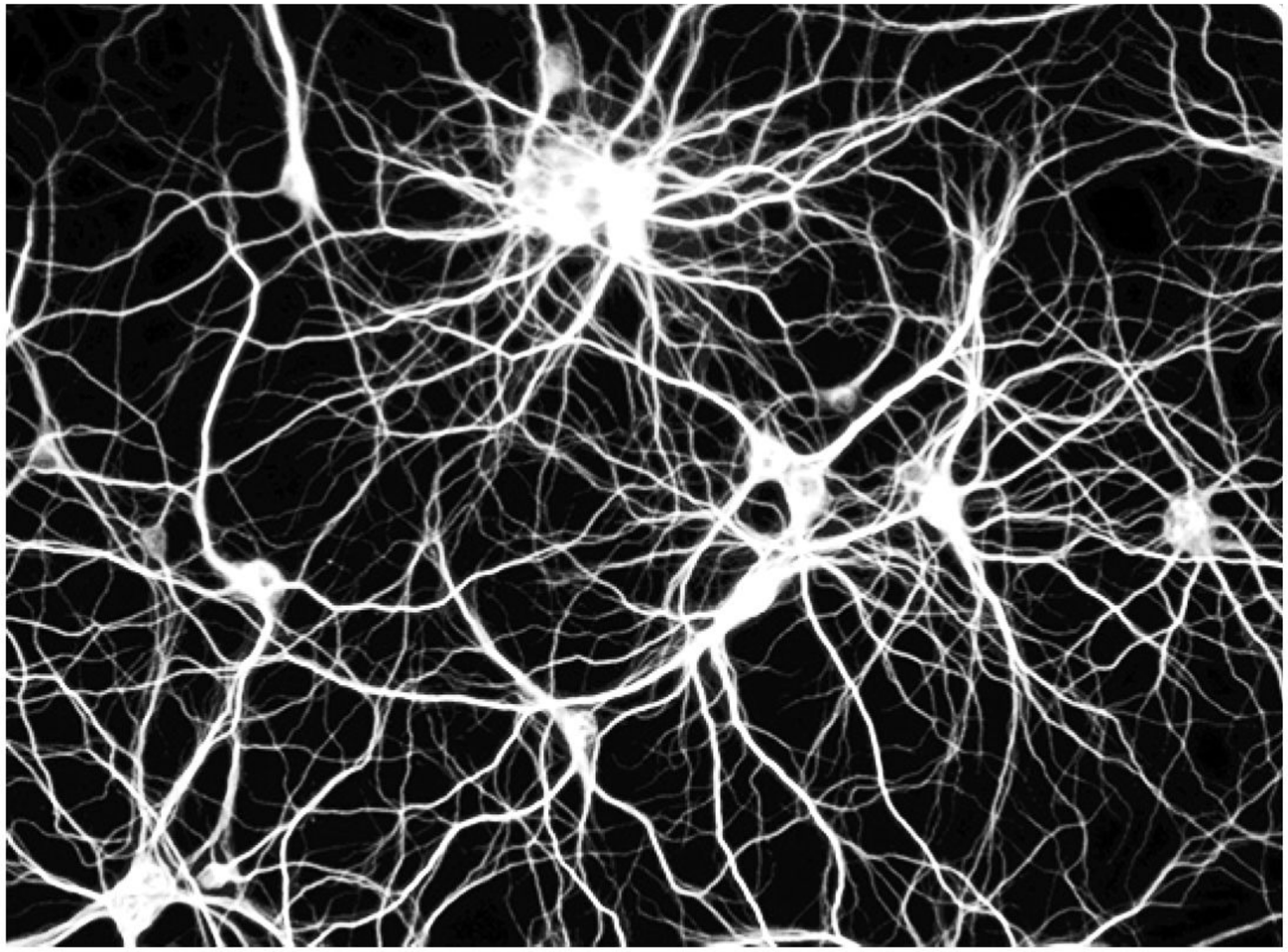
Fractals in Nature



All living systems entail a fractal approach



Single cell (Purkinje cell)



Brain network

All living systems entail a fractal approach

Besançon, France

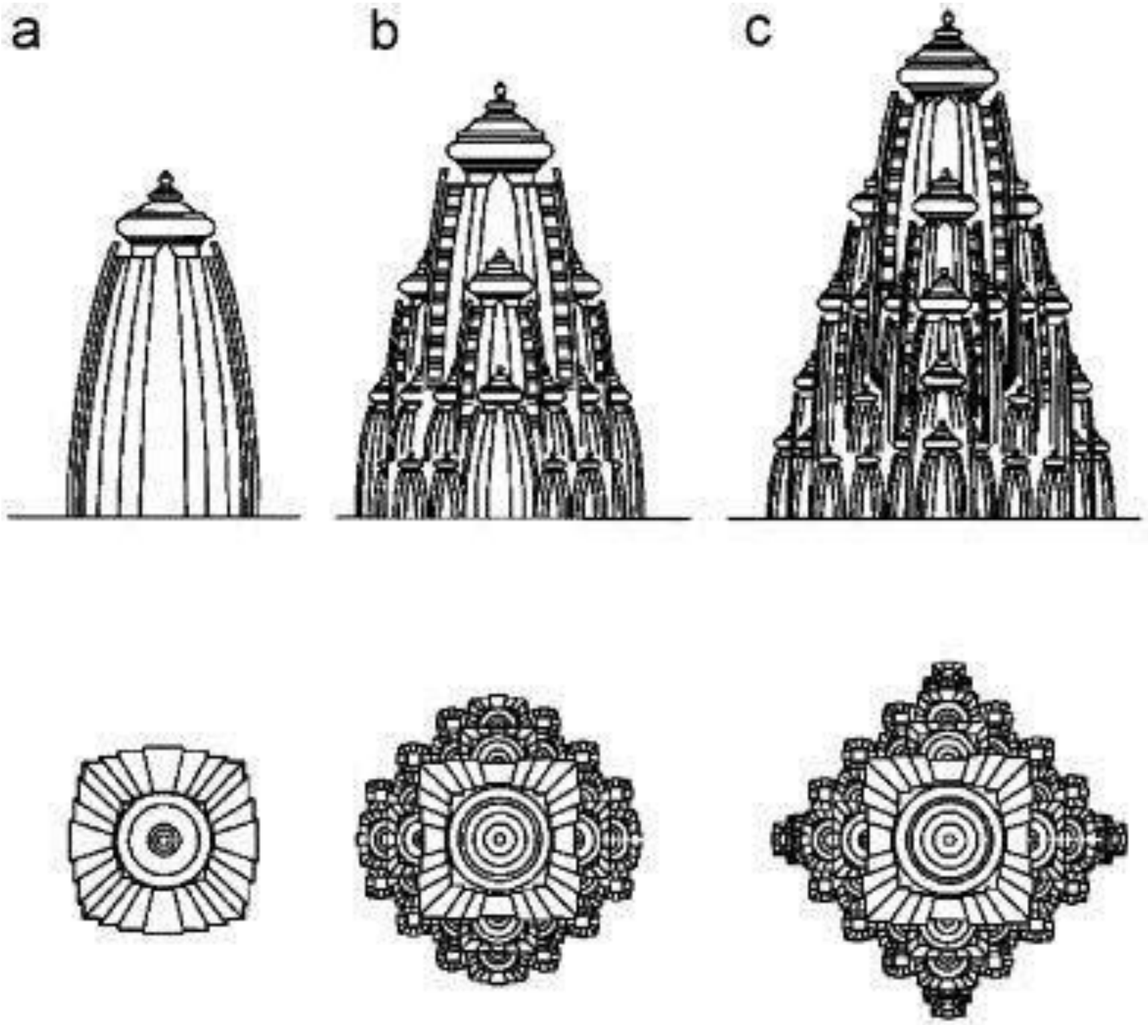
France/Belgium/ The Netherlands (NASA image)

Urban patterns are structured according to fractal laws (Hierarchy)

Paris, France

Hilbersheimer process-oriented plan for Seattle, 1957

Fractals in Architecture: Indian Hindu Temple



Biophilic design

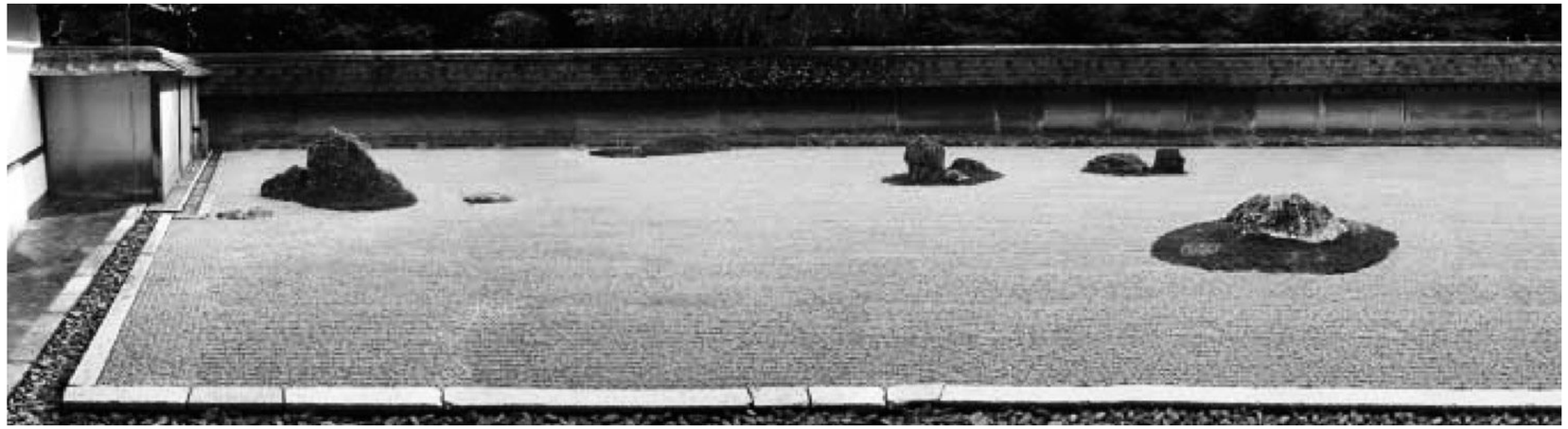
Fractals in Gothic Architecture

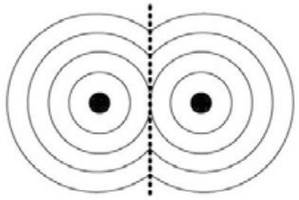
Fractals as aesthetic means in architecture

(a) (b) (c)

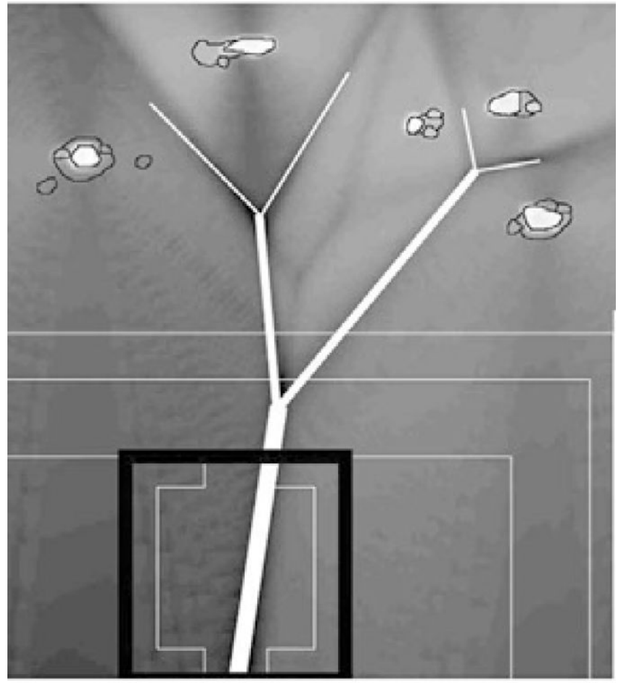
(a) Fractal floor design at the University of Oregon, (b) Wall paper in a state hospital in Graz, Asutria, (c) computer screen saver // Refreshing and relaxing. D = 1.6

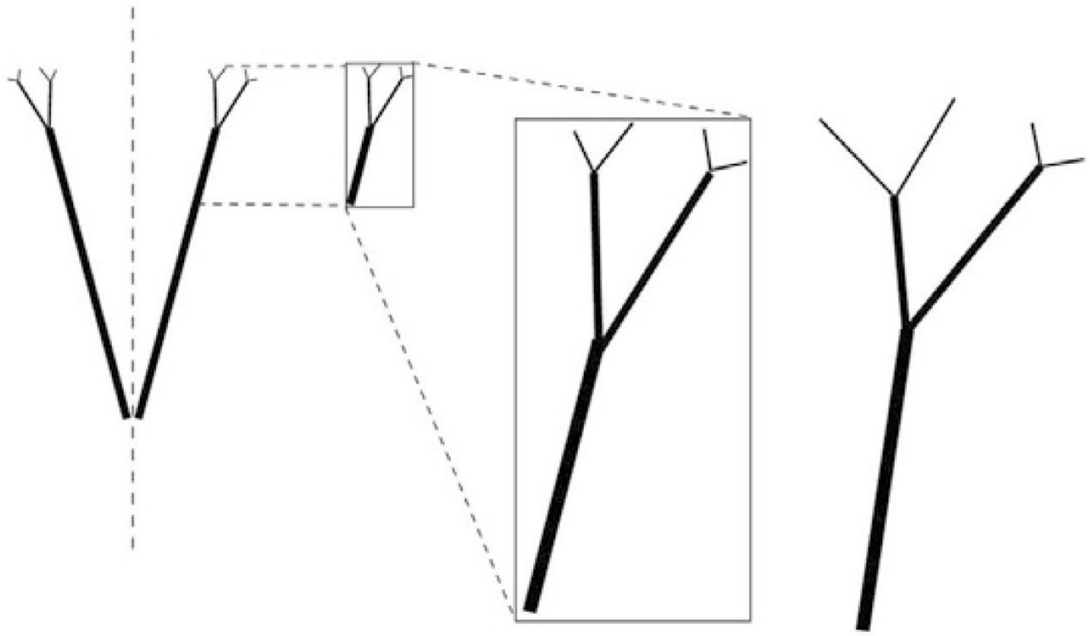
The fractal logic of a Buddhist Zen Garden in Kyoto (Van Toder & Lyons, 2005)





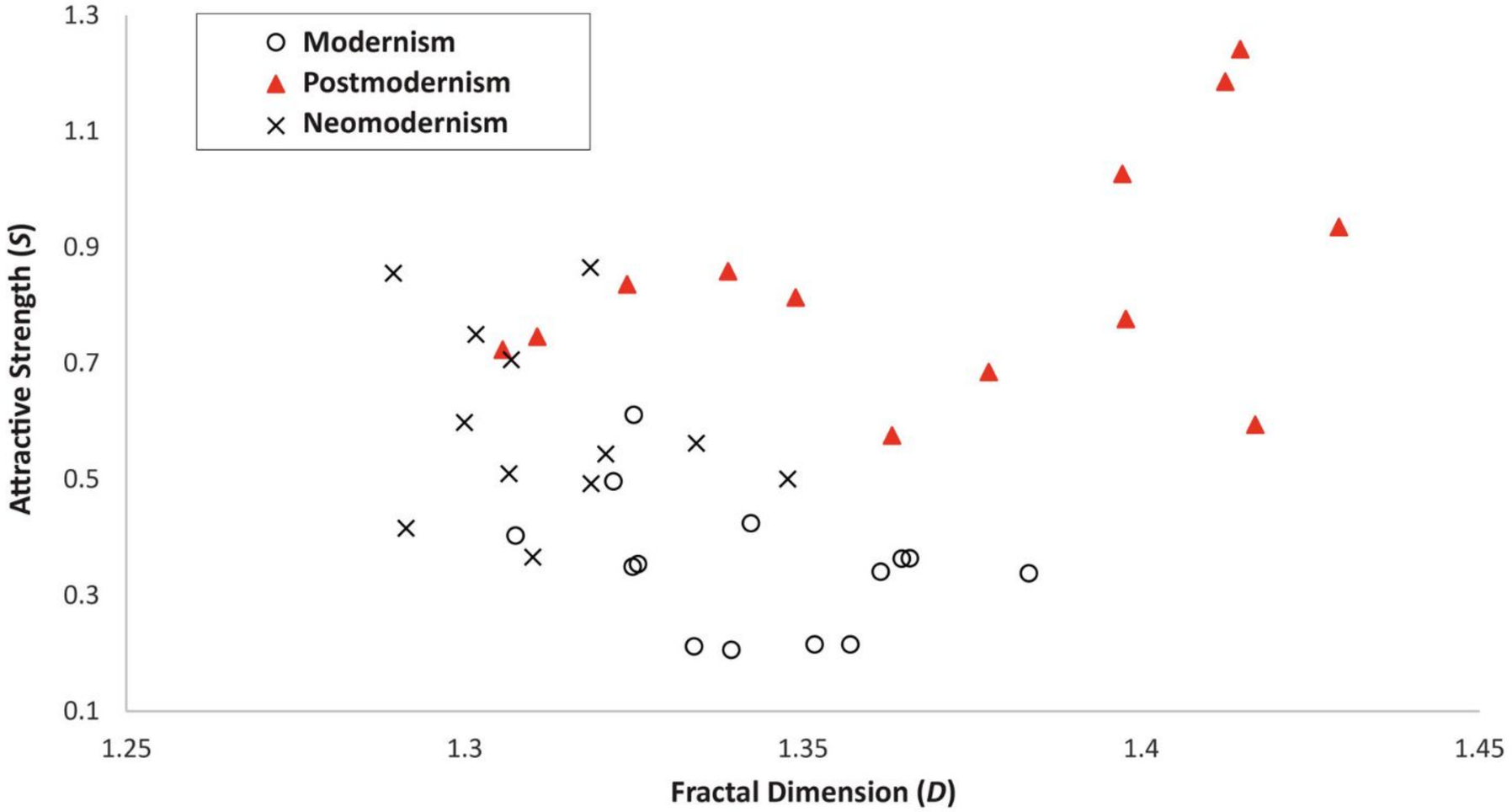
Media axis approach





The Fractal Dimension of Visual Attractiveness of Facades

Figure 6. D and S values of Neo-modern houses with their averages and ranges.

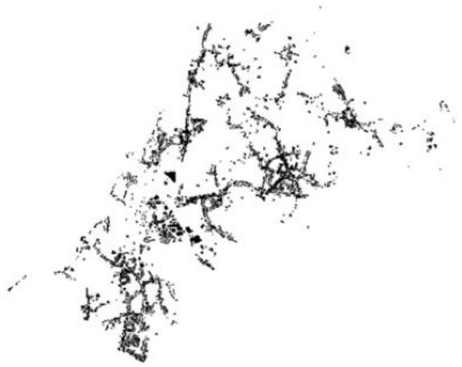


Postmodernism = mix of classical architecture and iconic detailing

Visual variety for place complexity of street scapes (Cooper & Oskrochi, 2008)

Tested 26 streets
(each street with 260 images)
with 31 participants.
D= 1.71

Quality of the built environment (Thomas et al. 2008)



Cluster 3: Blégny

Cluster 4: Rendeux

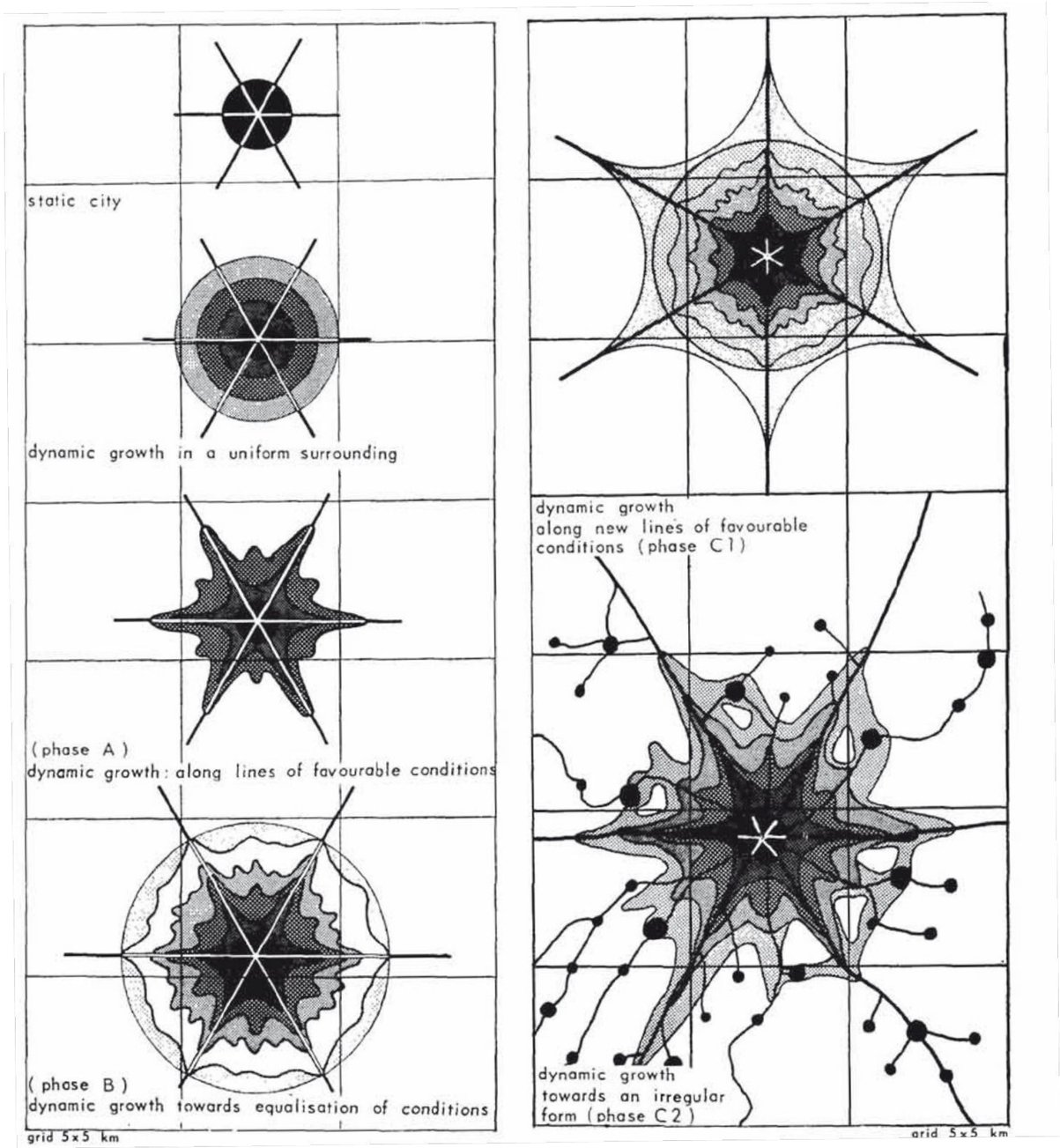
Cluster 5: Chaudfontaine

Cluster 6: Ciney

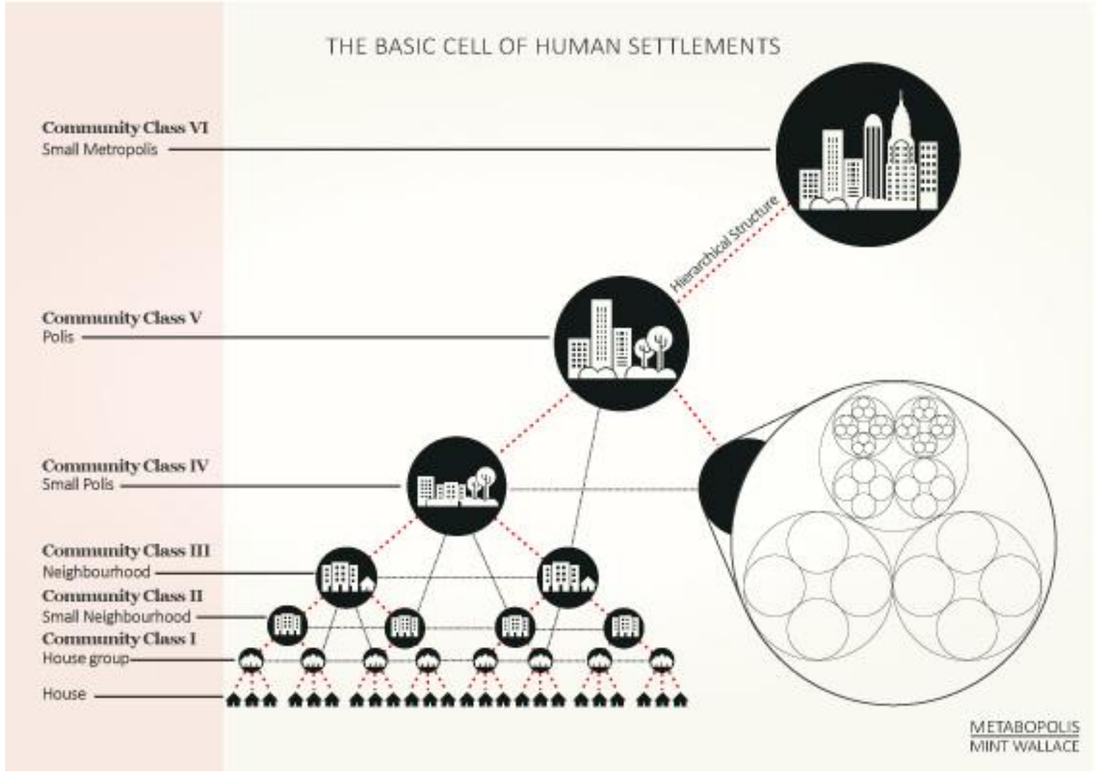
It has been recognized that fractal cities are directly linked to the satisfaction of residents and their environment followed by a distribution of functions. This study proved a theoretical and statistical relationship between fractal indices and the quality of the built environment (Thomas et al., 2008).

PART II: A BRIEF HISTORY OF THE FRACTAL CITY

Constantinos Doxiadis (1942, 1968) Ekistics

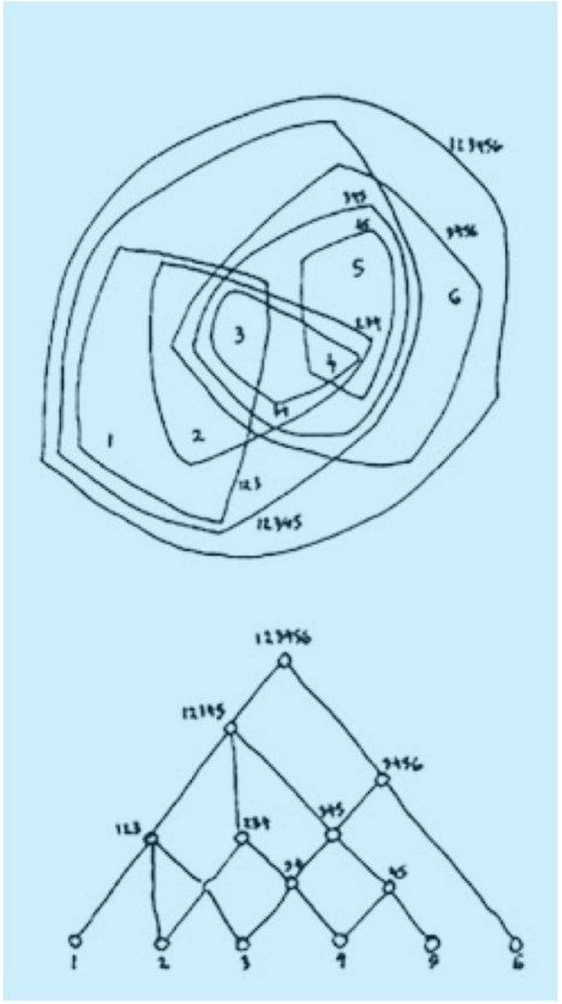


- Minimizing effort in terms of time, energy and cost.
- Maximizing of human potential – certain locations permit maximal contacts.
- Optimization of men`s relationship with nature

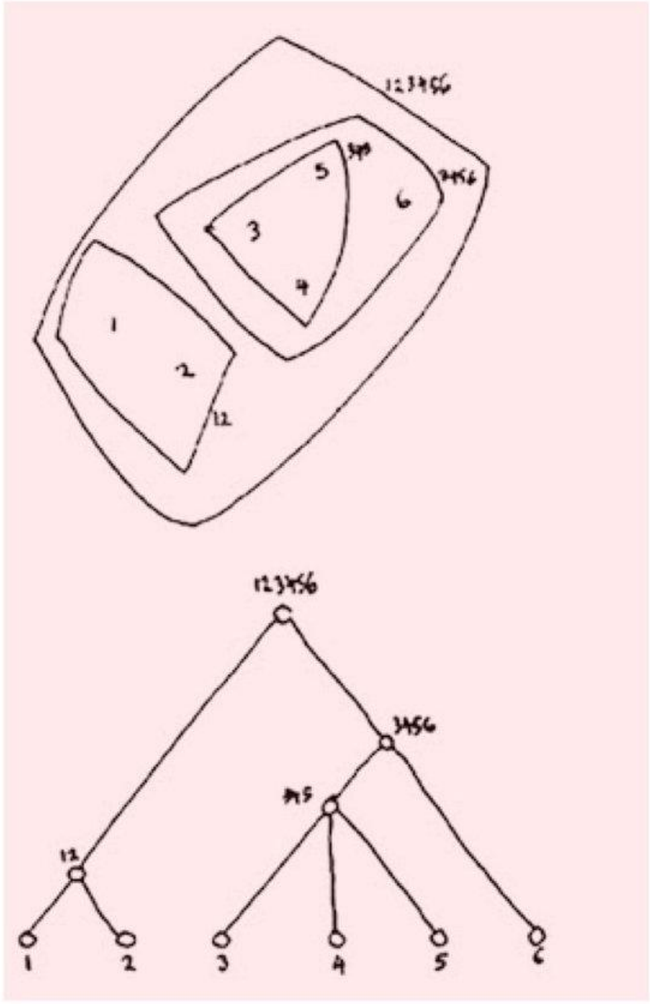


Christopher Alexander (1965) – A city is not a tree

This is a semi-lattice structure.
Here, **elements within a system interact with each other** with no defined hierarchy.



This is a tree structure.
It doesn't mean a plant, it means **a way of structuring things in a branching way.**



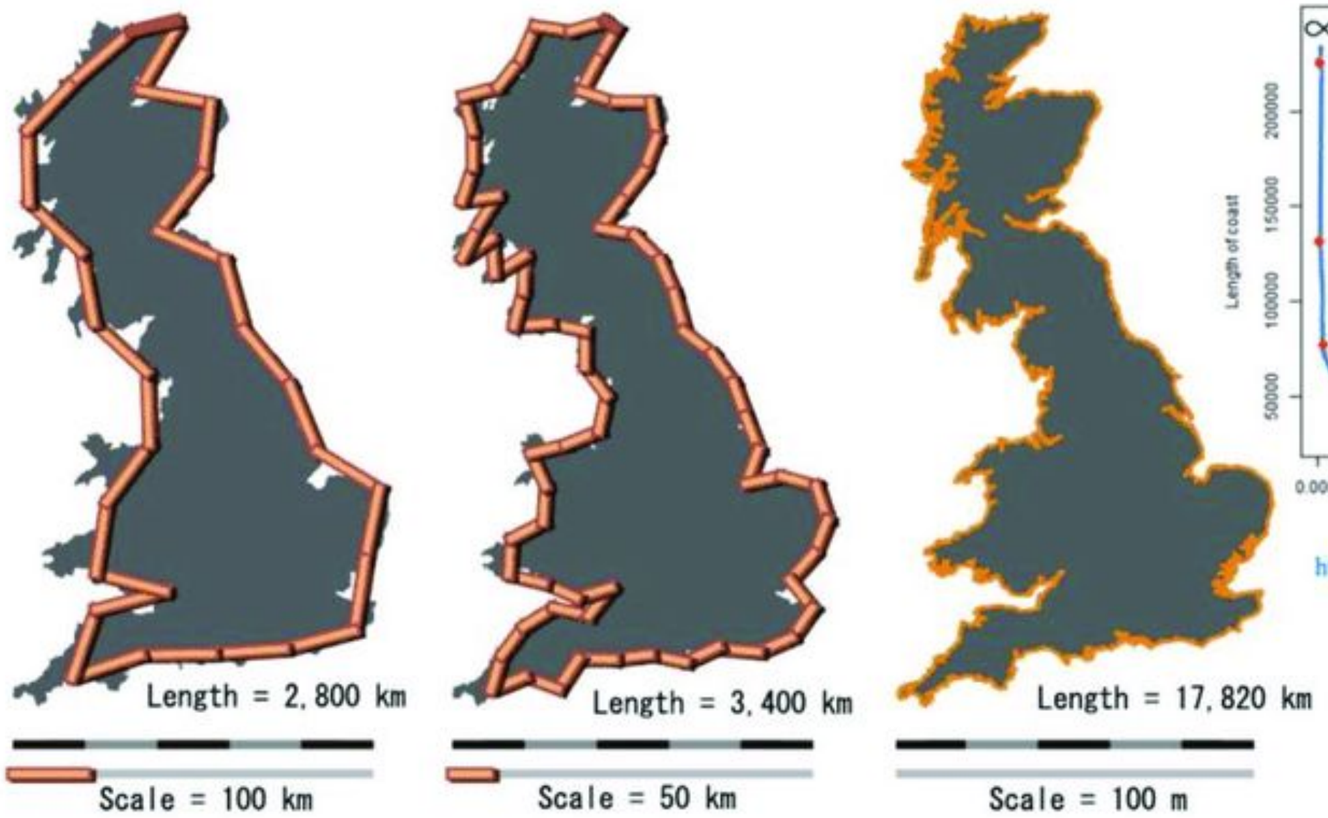
- Natural cities that have emerged over time consist of a semi-lattice structure and thus incorporate a more distributed street network.
- Cities with a semi-lattice structureusually have a complex transportation network, and their functions are well integrated.
- According to Alexander, this structure improves a city's economic development, its safety, and its liveliness.
- Thus, a distributed street network encourages a mixture of functions and movement patterns, while a non-distributed street network promotes the opposite.
- Highlighting the difference between a distributed and non-distributed grid on a map indicates the degree of complexity of movement routes for a city.

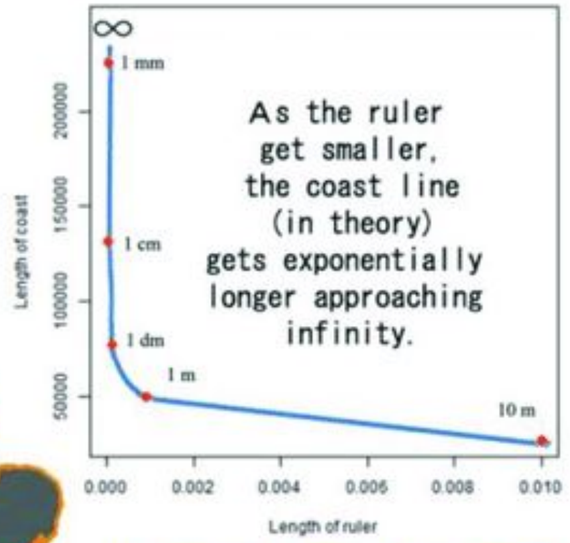
Christopher Alexander – A pattern language

At the core […] is the idea people should design their homes, streets, and communities. This idea […] comes from the observation most of the wonderful places of the world were not made by architects, but by the people.

—*Christopher Alexander et al., A Pattern Language*

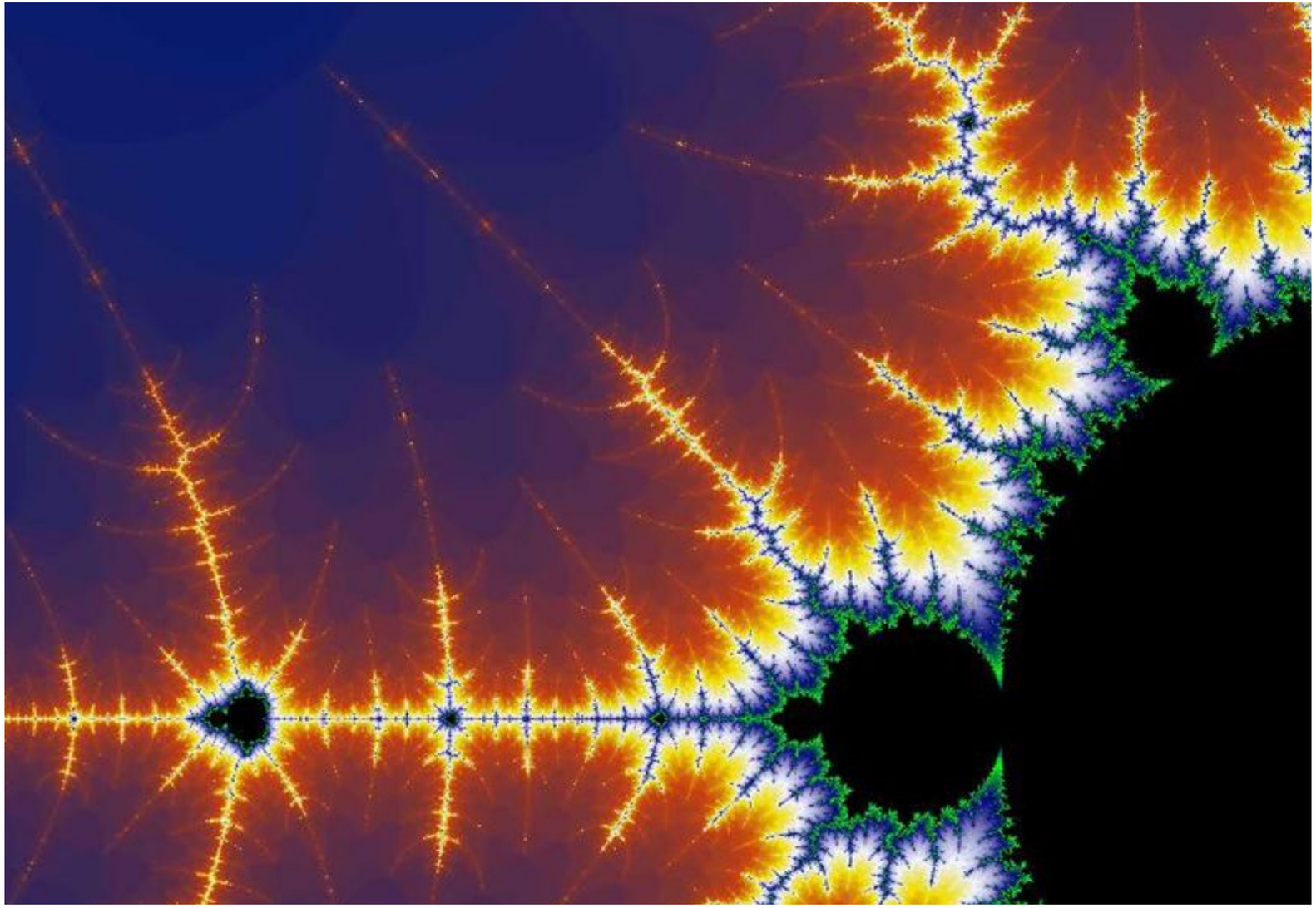
Mandelbrot (1977) How long is the Britain`s coastline?



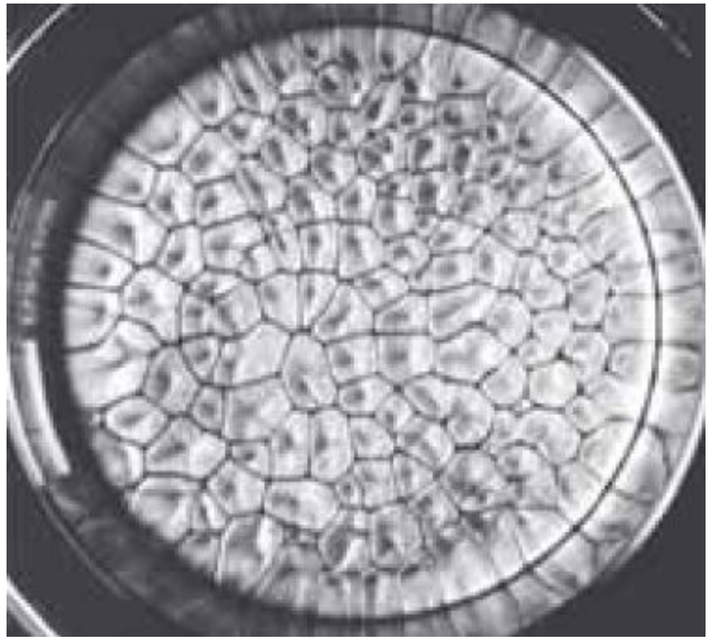


http://rspatial.org/cases/rst/2-coastline.html

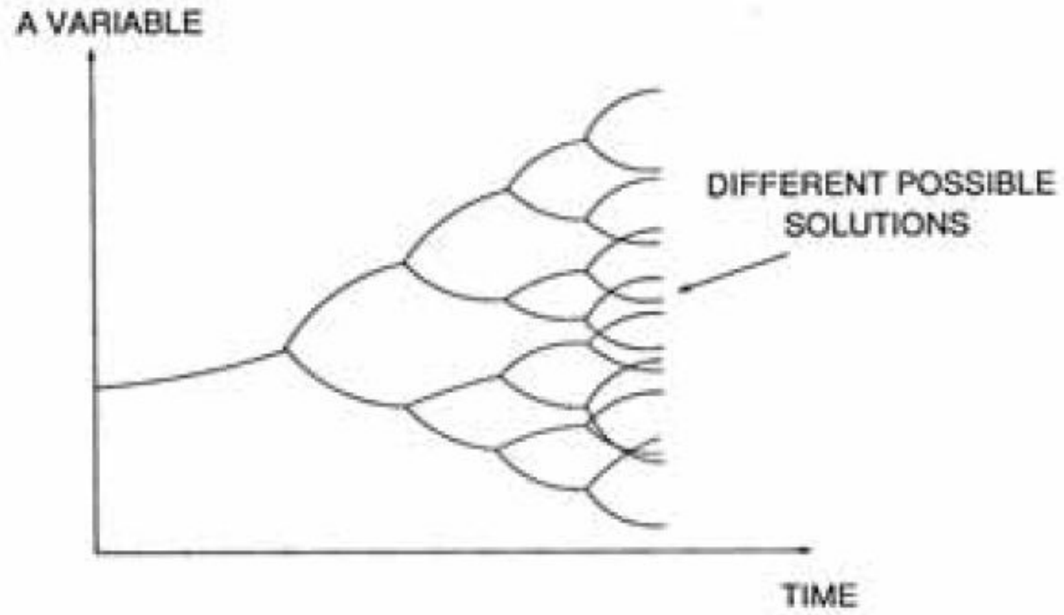
Mandelbrot (1977) – a new mathematical language



A system of self—organization in thermodynamics (Prigogine, 1977)



Bernard cells

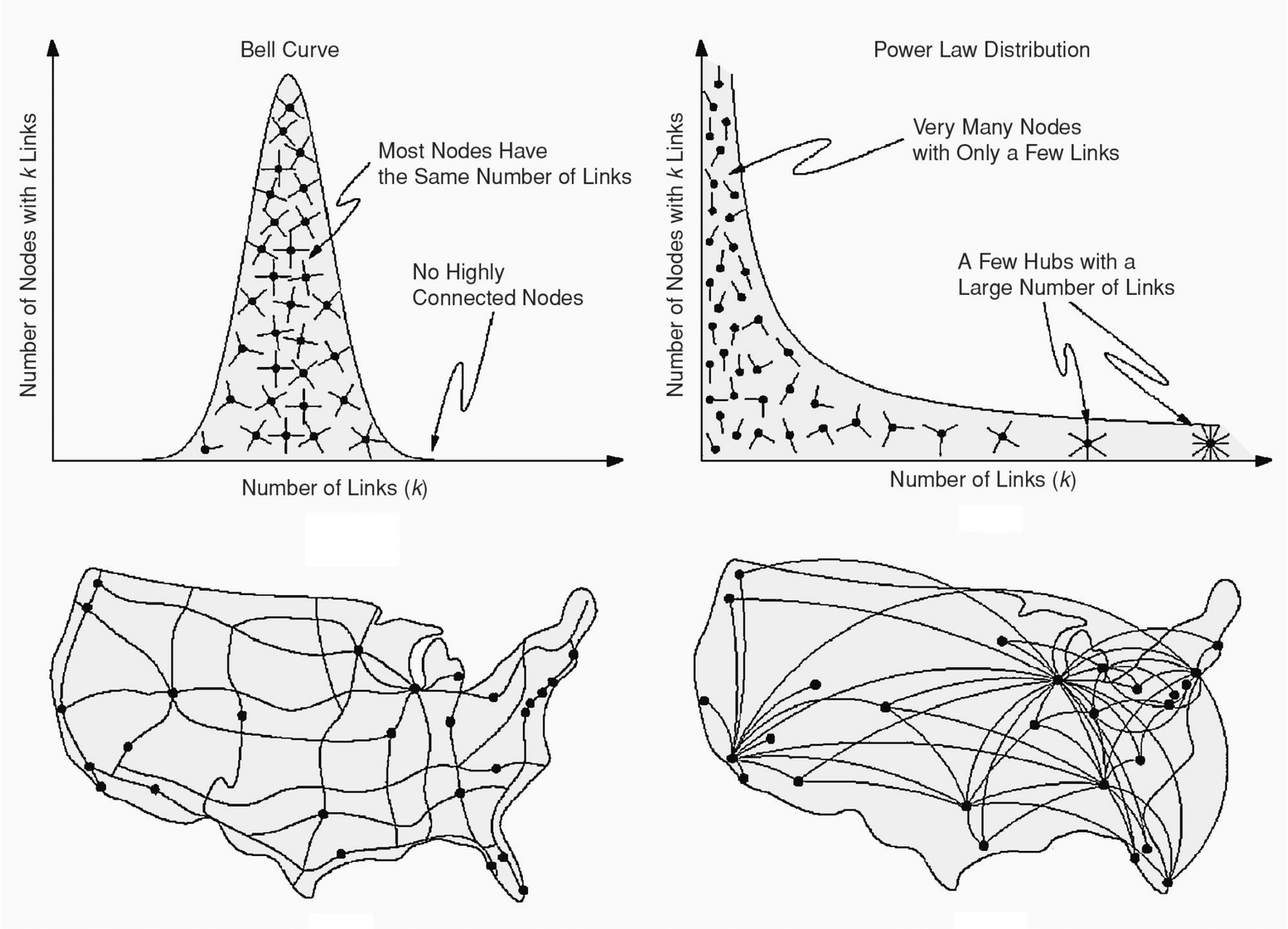


Bifurcation tree

The nobel prize laureate Prigogine outlined self-organisation for urban settlements in his conception of dissipative structures, as the structure of a city can only be understood with regard to its economic exchange with its hinterland (Prigogine, 1980, 12). Not only is the exchange with its
hinterland of importance, but also the activities of each actor within the system.

Each actor may take simple decisions, but from a global view the dynamic flux - all decision criteria of all actors - unfold a complex, dynamic system of the self-organisational principle. Hierarchical street patterns, centres and sub-centres - the general hierarchy of a city - is the result of all actors´ individual decision criterias of movement.

The idea of power laws in nature (Barabasi, 2003)



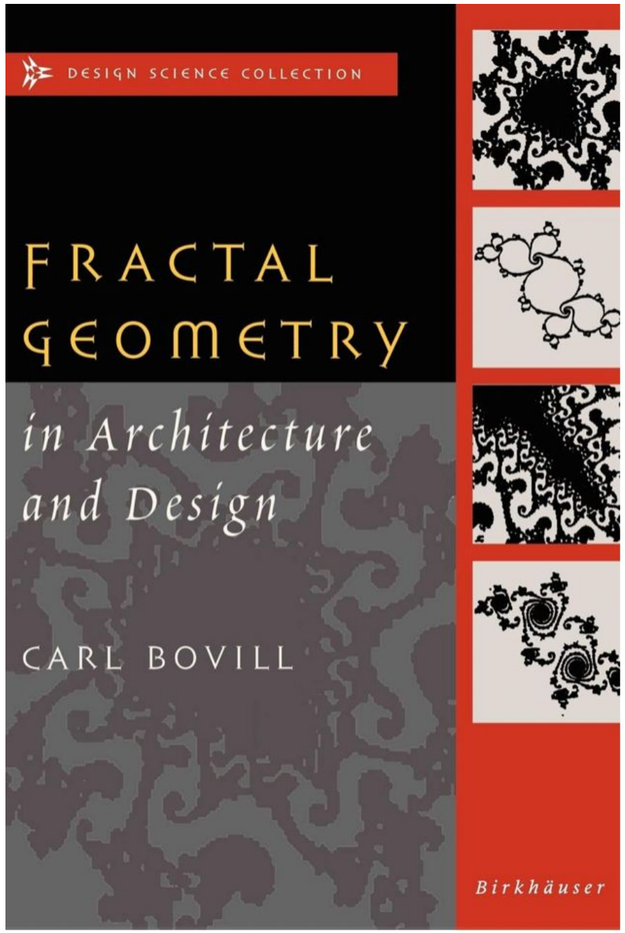
Power laws => interrelated, functional efficiency; a functional cause feeding a structural system.

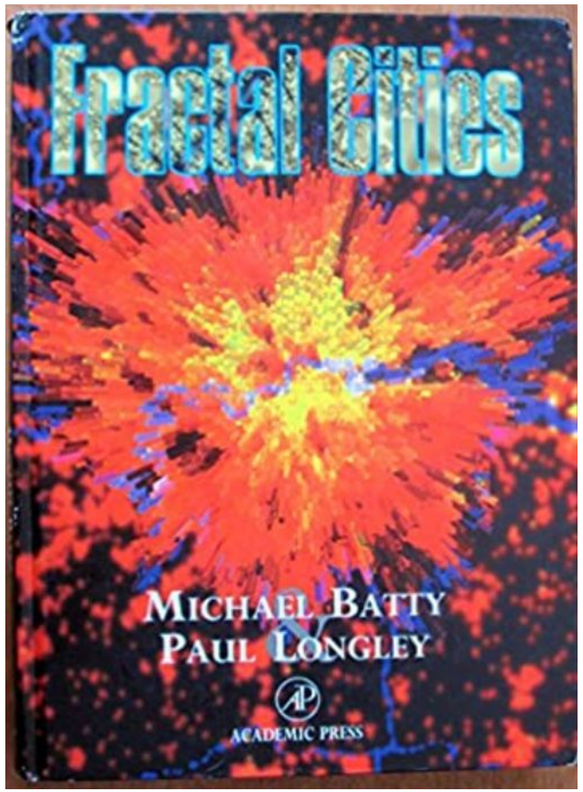
Only a few applications for conceptual models

Architecture
Bovill 1996, Cooper 2000, Durmisevis 1998, Lorenz 2004,…

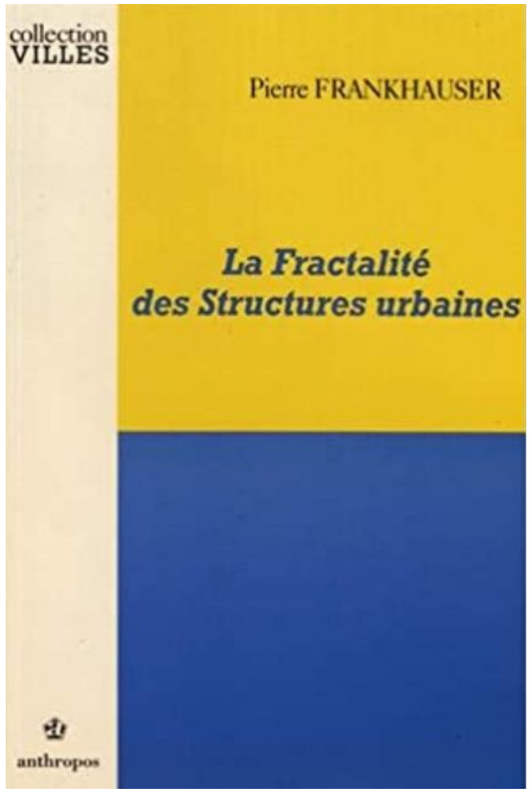
City scale
Frankhauser 2002, Salingaros 1999, 2003, 2006; Stamps, 2002, Thomas et al. 2013, Yamu and Frankhauser 2012, 2015,…

Regional scale
Yamu and Frankhauser 2015, Frankhauser et al. 2018,…





Mike Batty´s “Fractal Cities”
in 1994



Pierre Frankhauser´s
“La Fractalité de Structures urbaines” in 1994

– Early publications have shown that cities share fractal properties:

Arlinghaus 1985, Batty & Longley 1986, Fortheringham et al. 1989, Frankhauser 1988, Mandelbrot, 1982, White & Engelen, 1993, …

– A large number of contributions validate the hypothesis of cities as fractals:

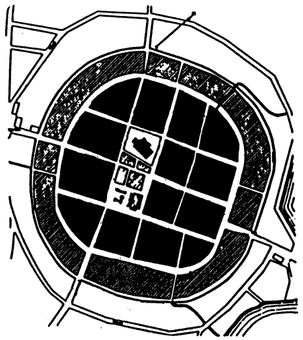
Batty & Longley 1994, Batty & Xie 1996, Benguigui et al. 2000, Keersmaecker et al. 2003, Longley & Mesev 2000, Shen 2002, Tannier & Pumain, 2005; Thomas et al. 2008,...

The main application of fractals are threefold

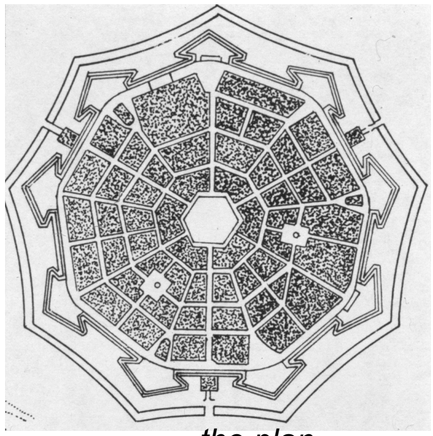
- as an analytical tool to understand spatial patterns,
- as a mathematical modelling approach to plan cities and regions, and
- as conceptual design approaches for the arrangement of elements for buildings.

Urban reference models

Friedeberg

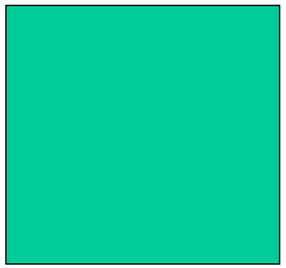


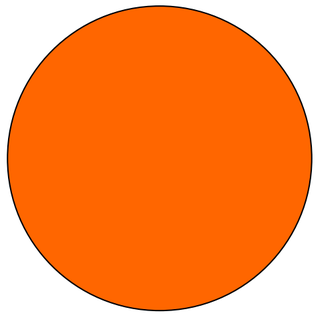
Palma Nuova

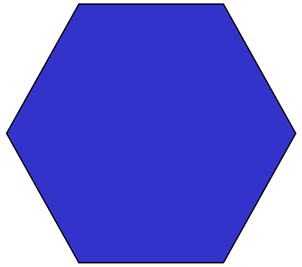


the plan ….

and reality





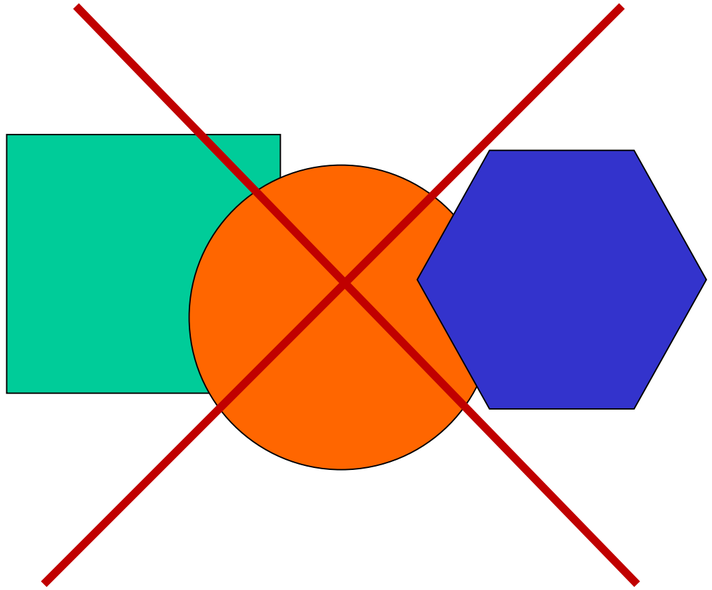


The geometric reference:
simple, regular, uni-scale.

PART III: UNDERSTANDING AND MODELLING CITIES AND REGIONS WITH A FRACTAL LOGIC

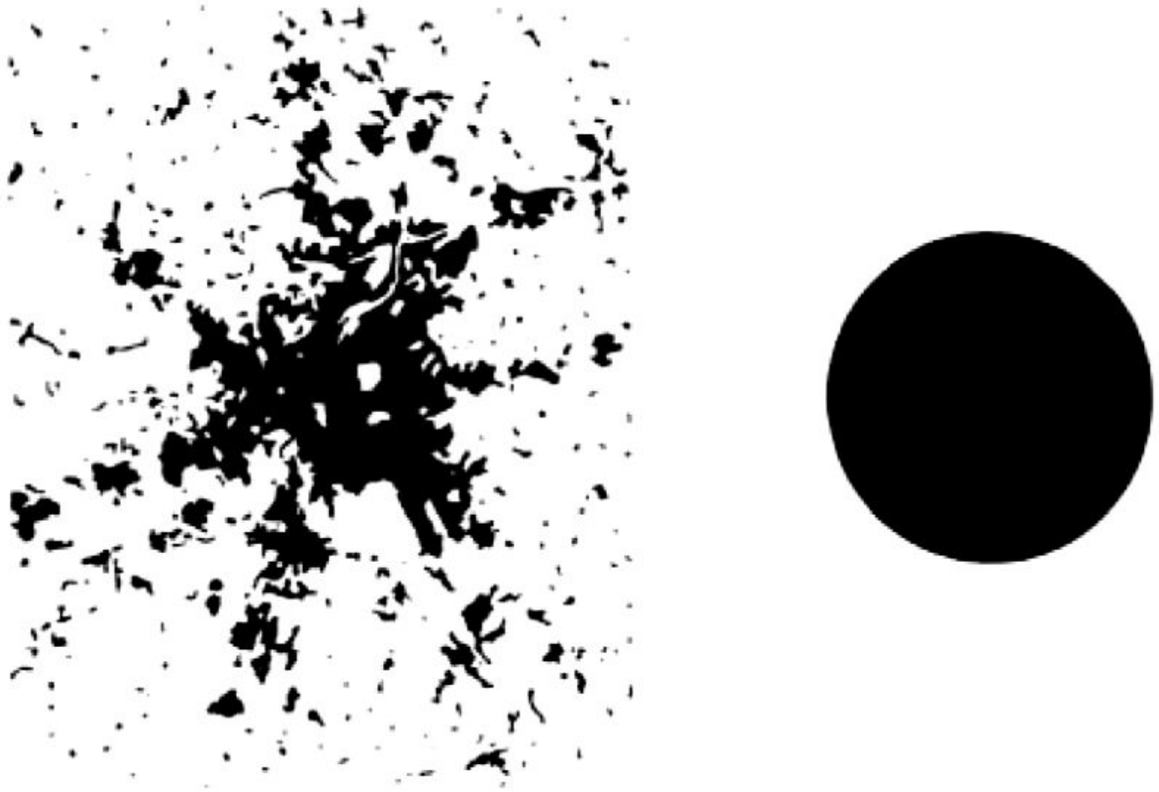
We cannot sufficiently describe complex metropolitan spatial patterns with a simple geometry

The build-up area of Berlin

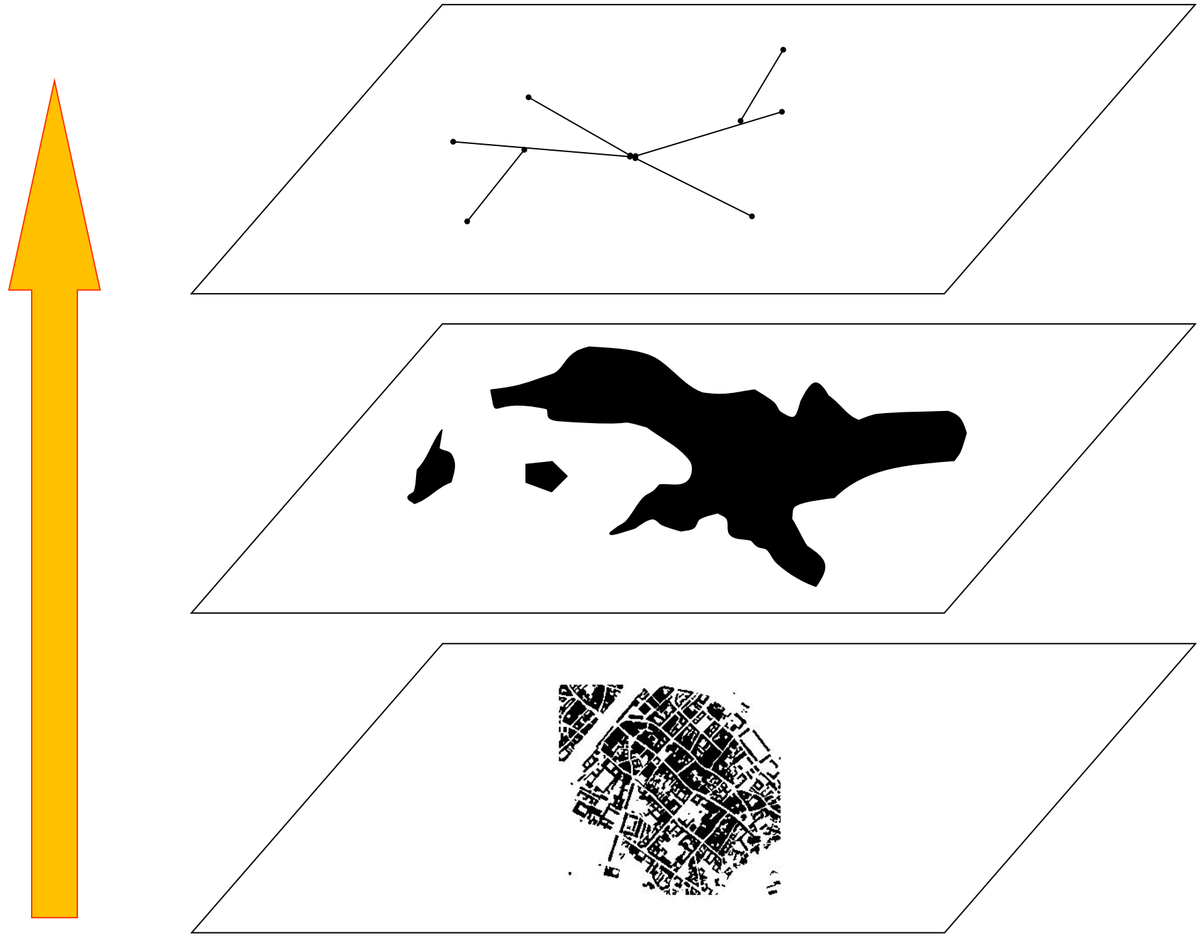


- complex – multi-scale
- perceived as “amorphous”
- hard to control

Berlin`s figure ground plan and a circle have an equal area



People are scale-dependent and scale-independent at the same time



Urbanisation is a multiscale phenomenon

Complex interactions Microscale macroscale Self-organizing system

spatial structure

pressure groups
socio-economic dynamics

social groups,
social references

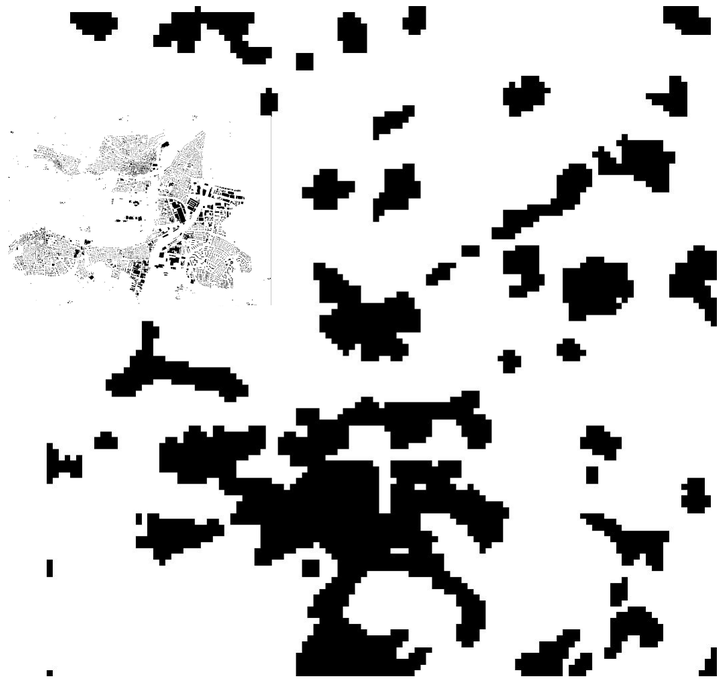
Individuals and households

Interaction ot cities on metropolitan scale

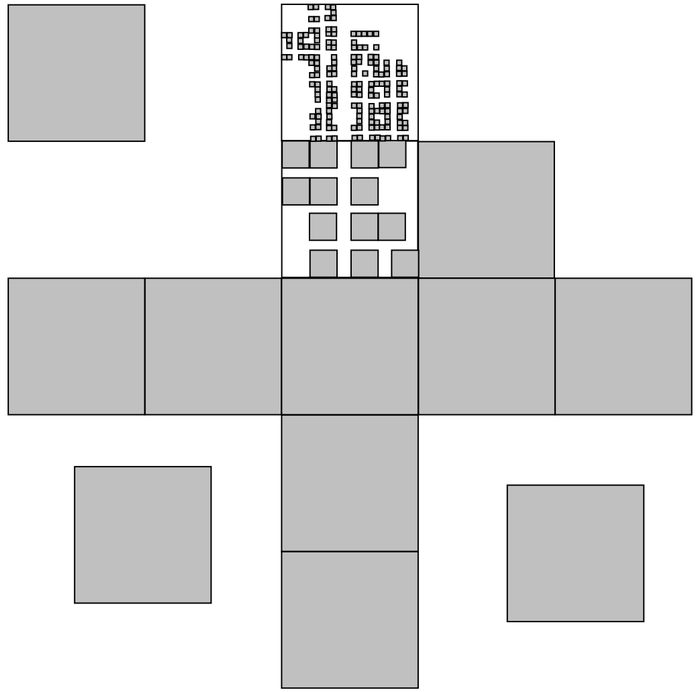
urbanism
urban scale

architecture
microscale

Metropolitan areas as multiscale phenomenon

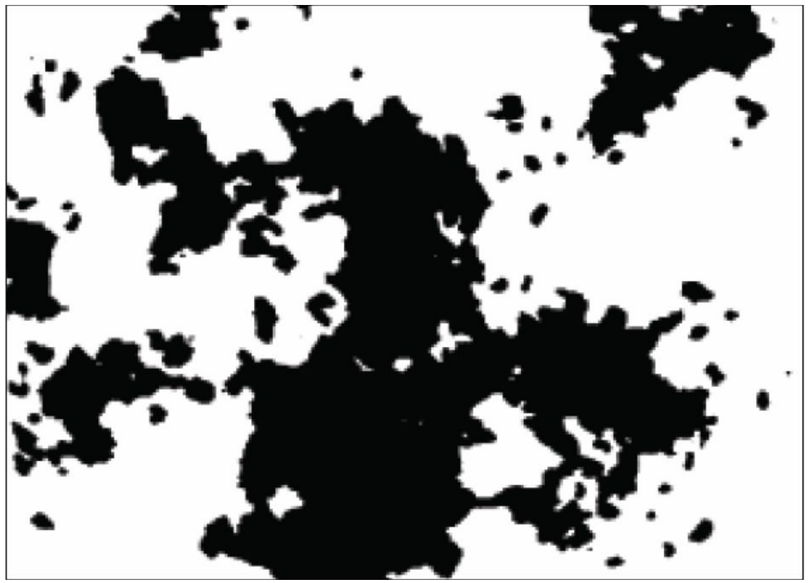


Zoom on the urbanized area of Stuttgart

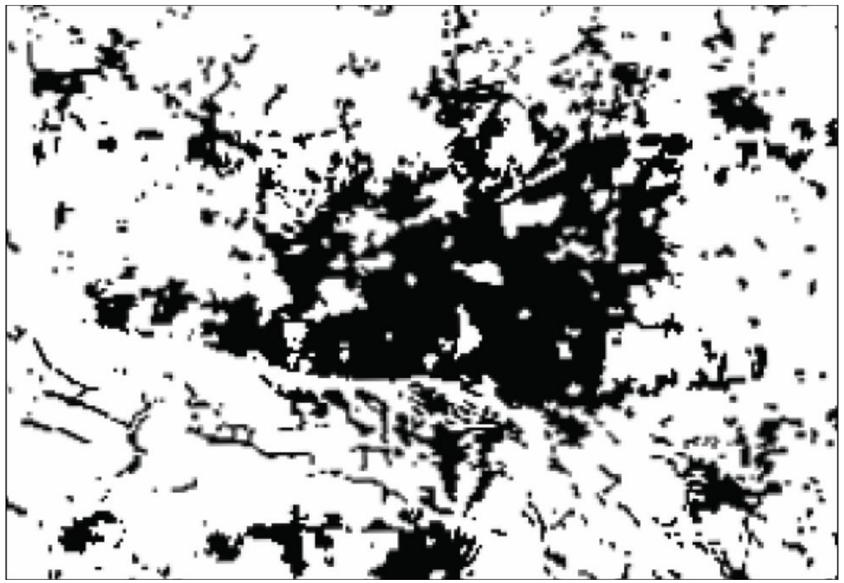


Zoom on a fractal structure

Morphological analogies



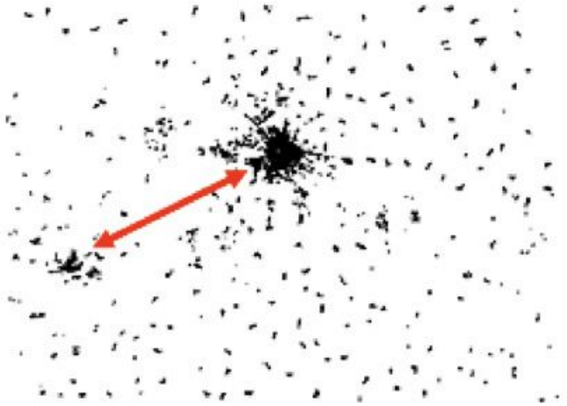
Fractal simulation



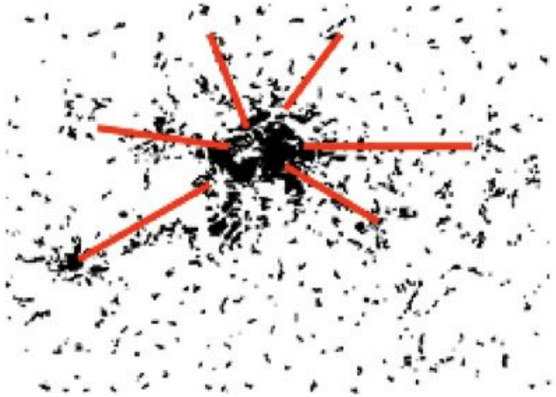
Hamburg

Berlin’s growth

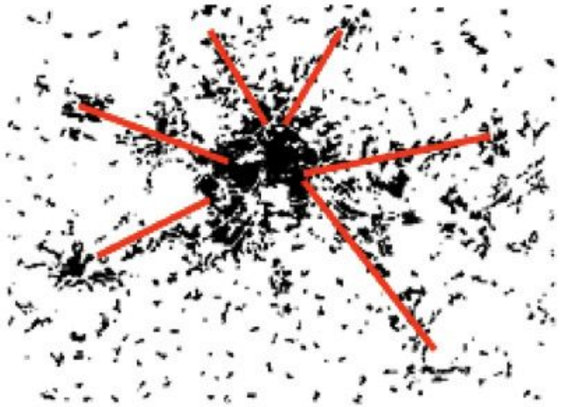
1875



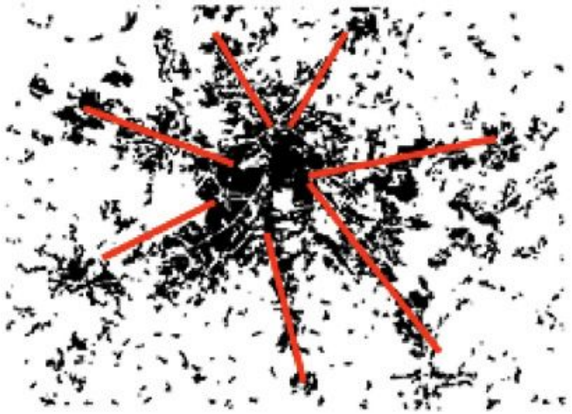
1910

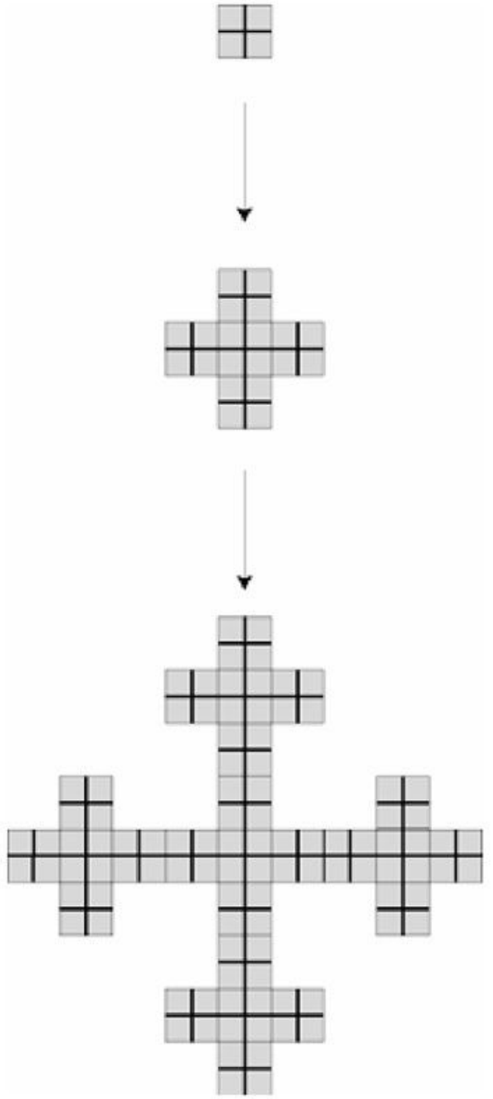


1920



1945

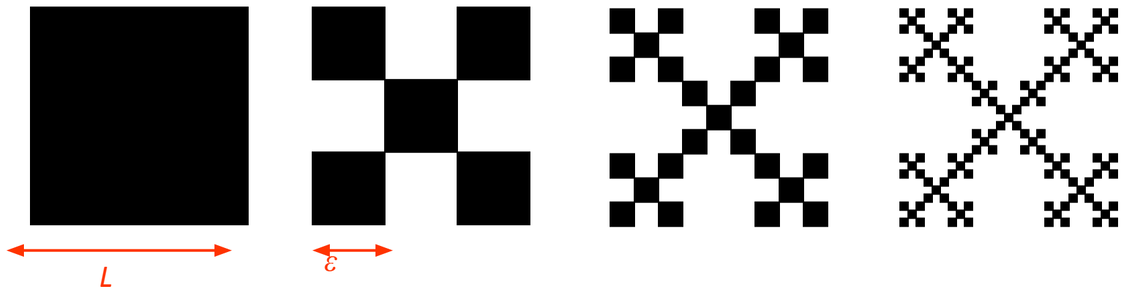




What is a fractal?

generation by iterative mapping

example : a Sierpinski carpet



initiator generator

$N = 5, \varepsilon = 1/3\ L = r\ L$

1st step	2nd step	3rd step
$N_1 = N$	$N_2 = N^2$	$N_3 = N^3$
$\varepsilon_1 = r\ L$	$\varepsilon_1 = r^2\ L$	$\varepsilon_1 = r^3\ L$

this scaling principle corresponds to a hyperbolic law

The fractal dimension

In course of iteration:

- Surface vanishes
- Border length tends to infinity

neither surface nor line nor points

In order to have a measure **fractal dimension**

fractal law

$$N_n \varepsilon^D = 1$$

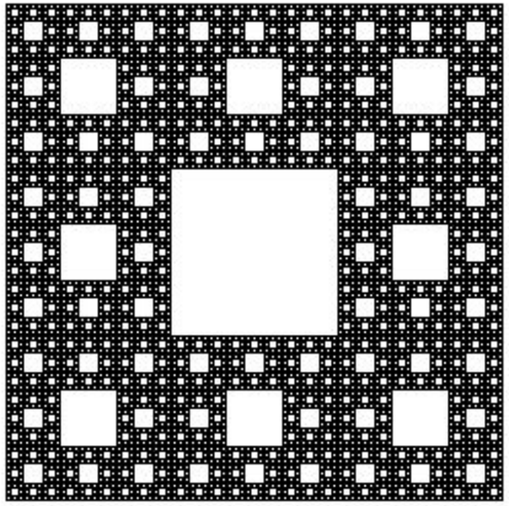
fractal dimension

constructed fractal:

$$D = -\frac{\log N}{\log r}$$

According to this definition for a surface there is *D=2* and for a line *D=1*

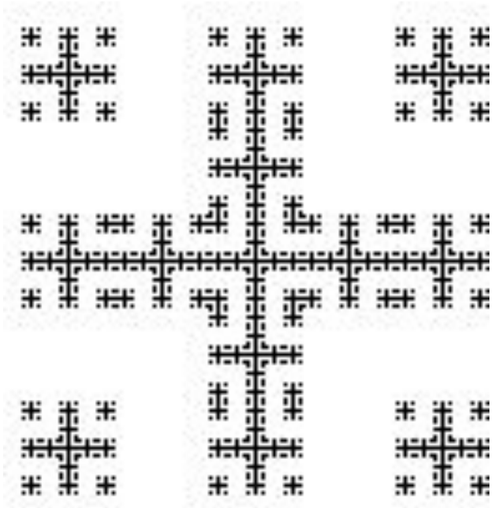
Every fractal shows a spatial hierarchy



N = 8, r =1/3
D = 1,89

Size/number relation follows a hyperbolical scaling law

Hierarchy of lacunas (gaps)

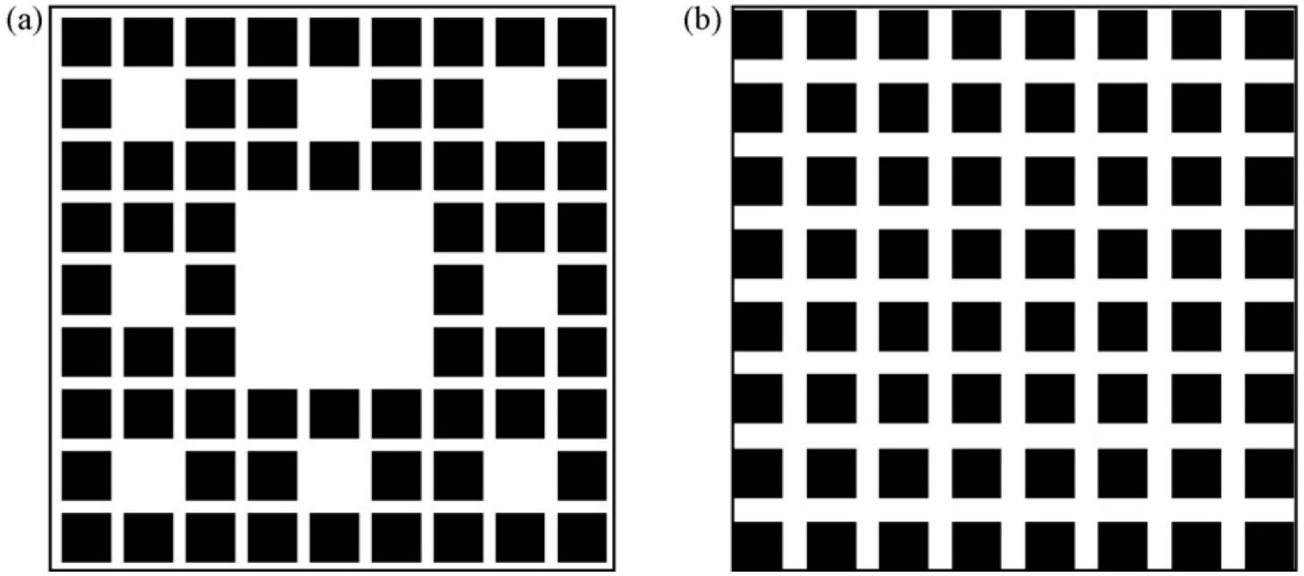


central cluster:
N = 9, r = 1/5
D = 1,37

total fractal:
N = 13, r = 1/5
D = 1,59

Hierarchy of clusters

The fractal dimension: Degree of uniformity in spatial distribution



Same number of squares – same surface of reference

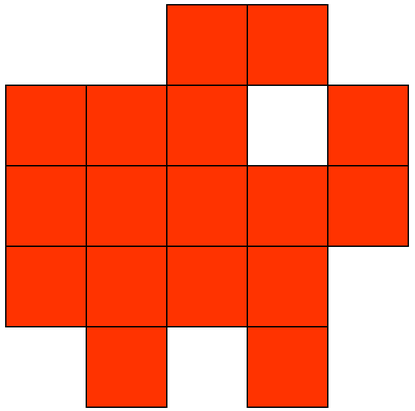
fractal distribution – uniform distibution

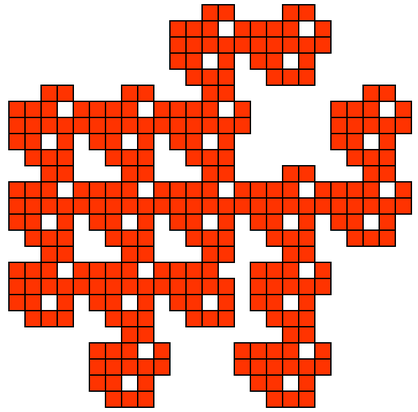
$D = 1,89$ – $D = 2$

– same density–

Random fractals for cities

A less symmetric generator …

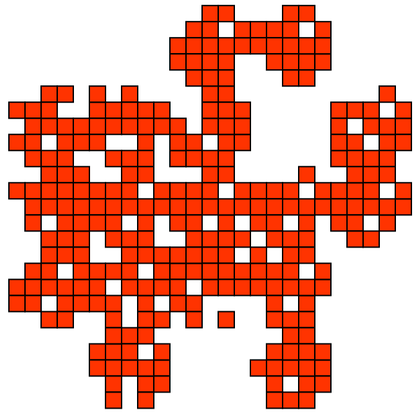




strict iteration

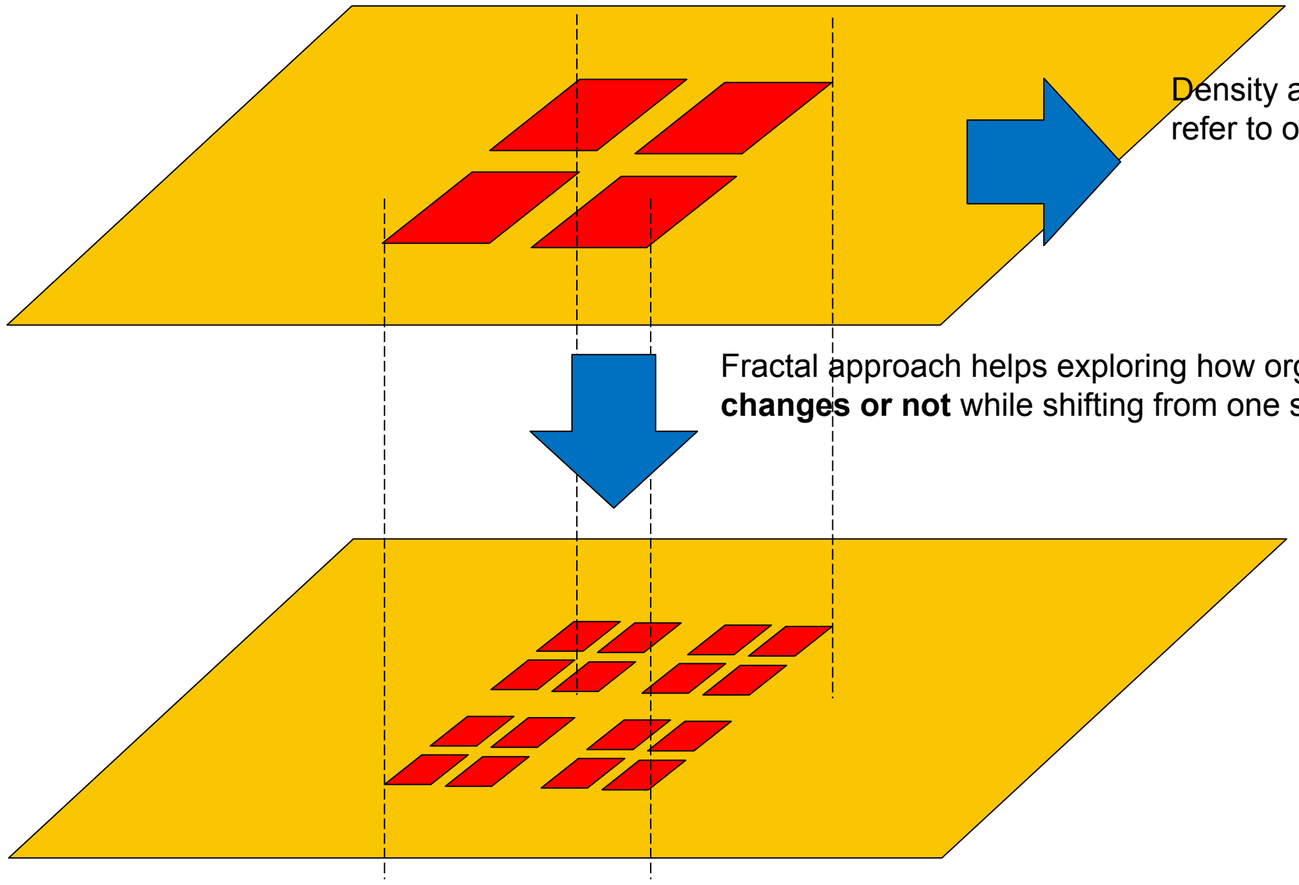
… and a more arbitrary position for the following steps

the already generated lacunes must be respected !!



random variation for 2nd step

Horizontal and vertical logic

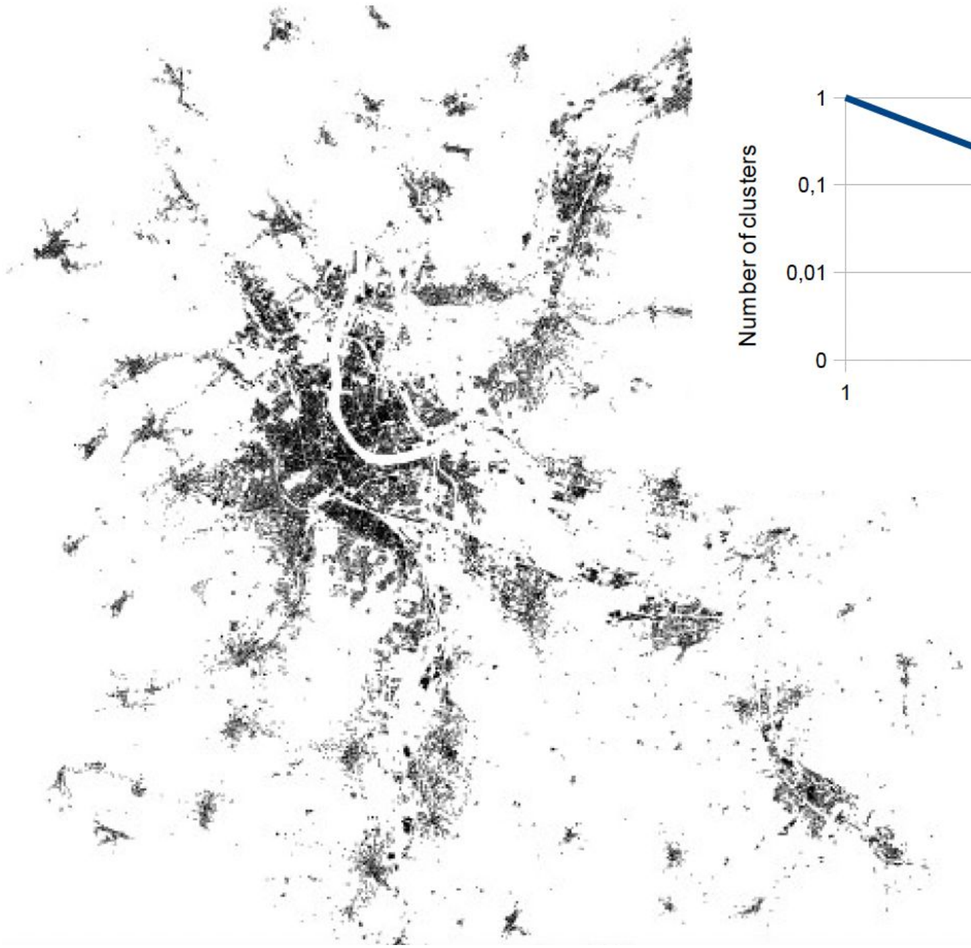


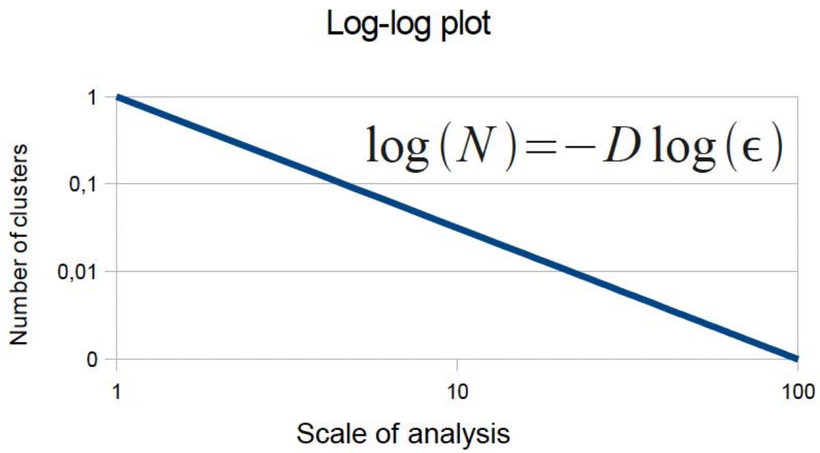
Density and related measures :
refer to one scale

Fractal approach helps exploring how organization
changes or not while shifting from one scale to the other

fractal geometry provides an interesting approach for describing and modelling complex forms like urban patterns

3 types of self-similarity



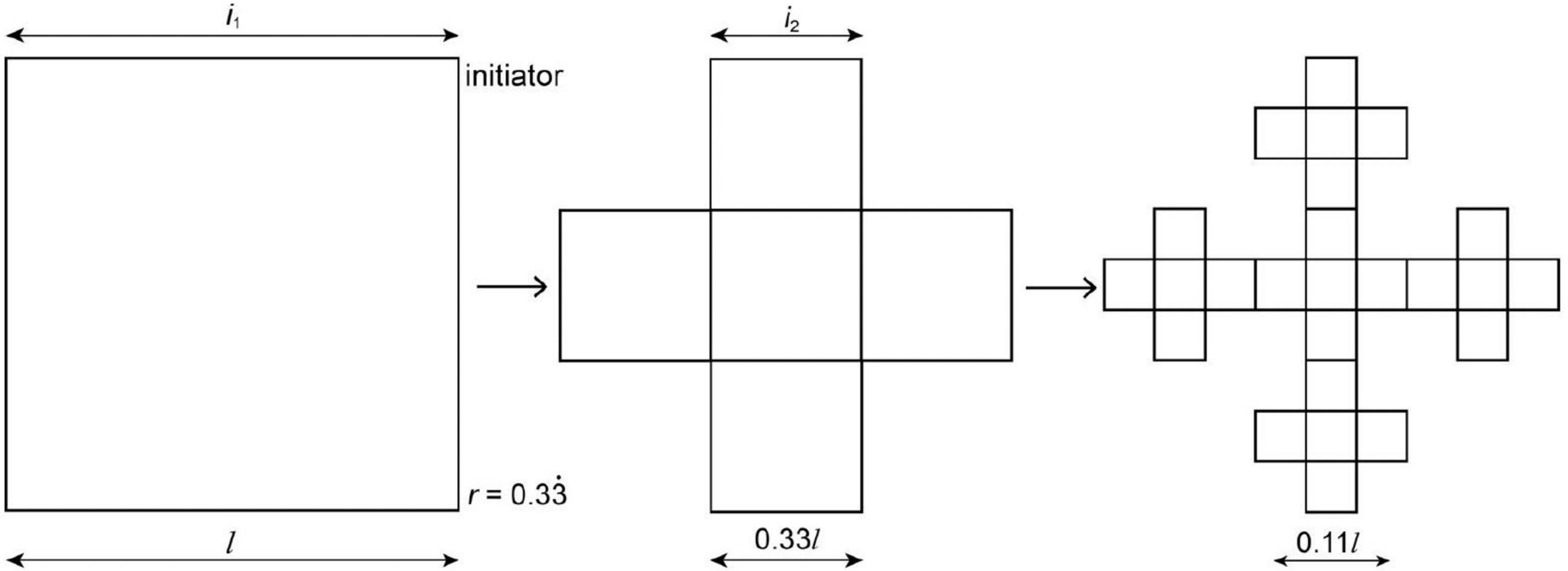


Basel in 1957

Fractal (correlation) dimension
D = 1.7

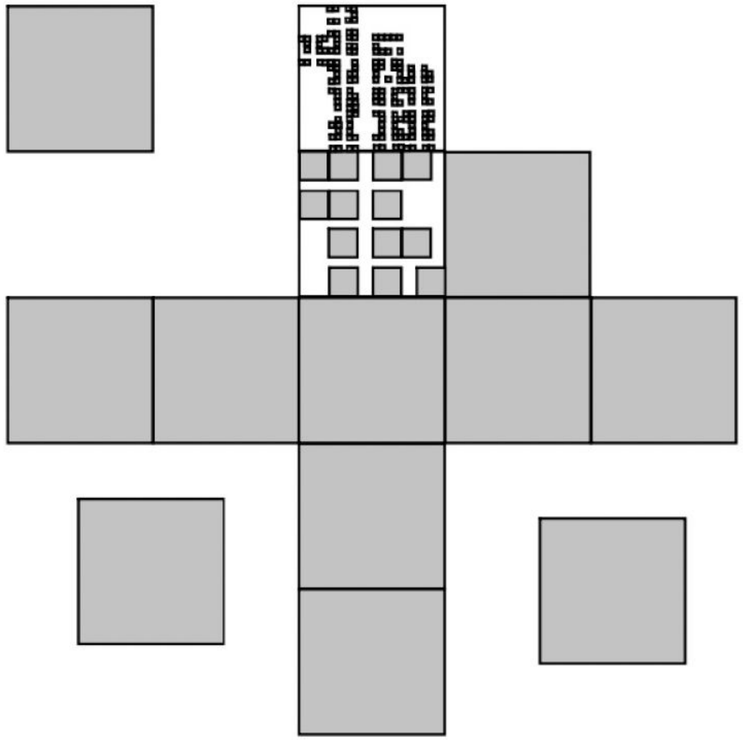
Statistical self-similarity

3 types of self-similarity



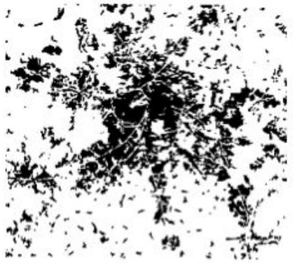
Theoretical self-similarity

3 types of self-similarity

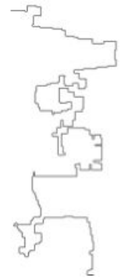


Pseudo self-similarity

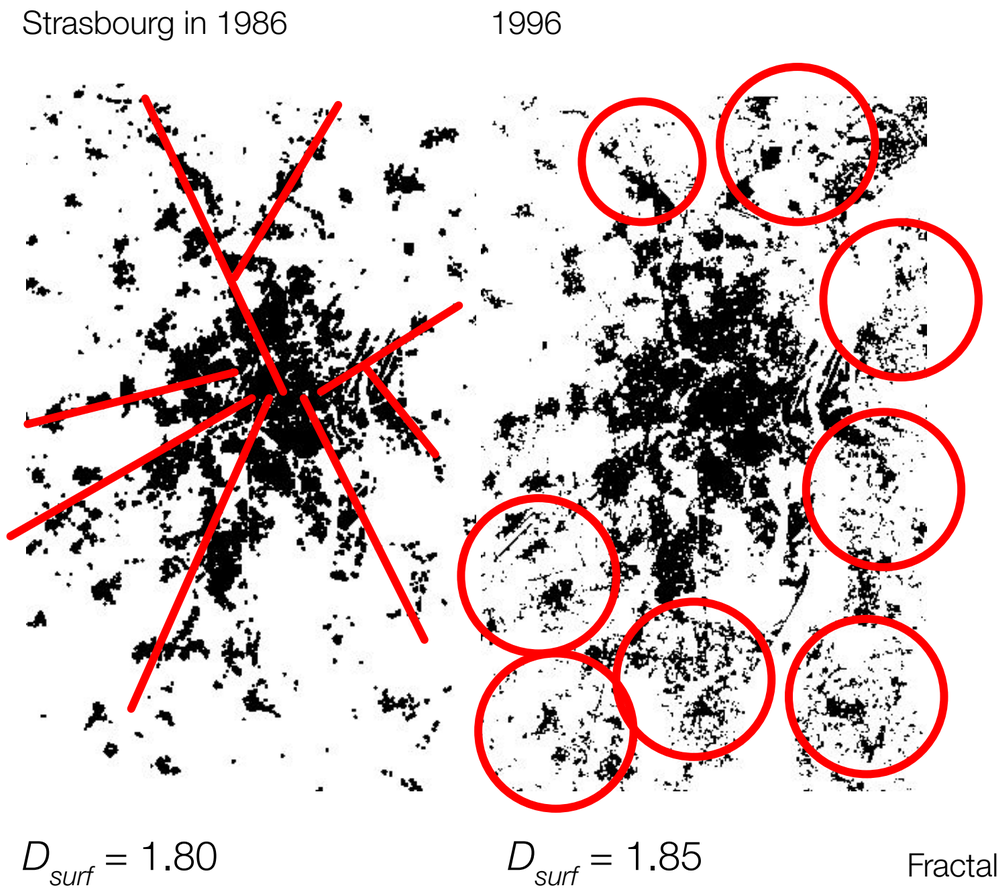
A classification of the fractal dimension

Application	Illustration	Scale	Fractal Dimension
Understanding the hierarchy of the spatial clusters on a metropolitan scale		Metropolitan	Fractal surface dimension D_{Surf} or number of spatial clusters
Understanding the non-uniform distribution of spatial elements of a given built-up area		Urban	Fractal surface dimension D_{Surf}
Analysing the differences of various spatial patterns following different distribution laws.		Urban	Comparison of different D_{Surf} values

A classification of the fractal dimension

Measuring the - fragmentation of urban fringes (boundary) from built-up space including spatial islands and non-built-up space inbetween.		Urban, Metropolitan	Fractal boundary dimension of spatial ensembles $D_{\text{Bord tot}}$
Measuring the tortuosity of the dominant aggregated boundary.		Urban, Metropolitan	Fractal aggregated boundary dimension $D_{\text{Bord ag}}$
Understanding where the network follows its own fractal logic and renders *access ability* (accessibility) of buildings. It is a complementary measure to D_{surf}.		Urban, Metropolitan	Fractal network dimension D_{Netw}

Fractals and transport



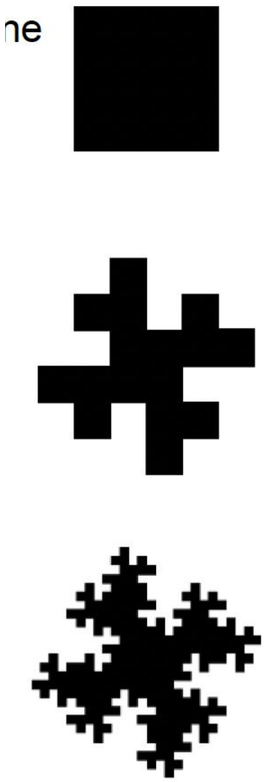
Fractality in cities corresponds to urban fabrics dominated by **public transport networks** and **pedestrian networks**. Through **motorisation** urban patterns become more **uniform**.

Accessibility and the multi-scale relations to the urban fringe in fractal models

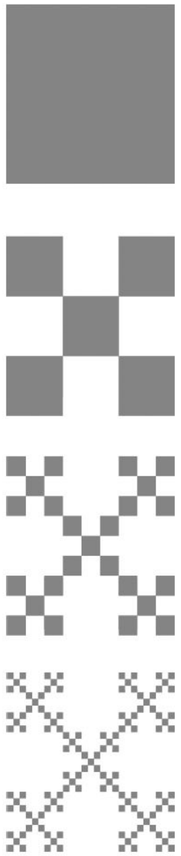
Teragone

Frankhauser 2000

Compared to a “square city”, the teragone does not consume more space; the mean distance to the town center is almost equal; the mean distance to the urban boundary is smaller.



Sierpinski

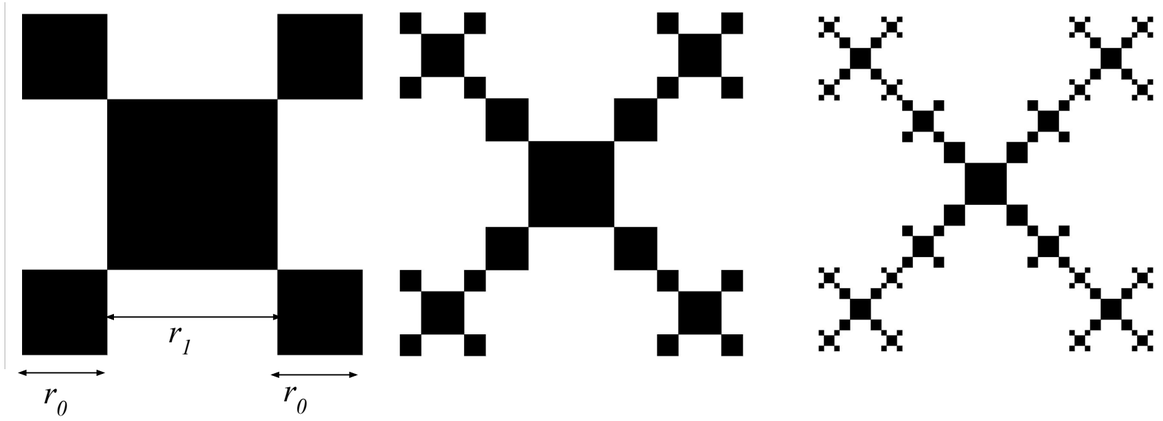


Frankhauser & Genre-Grandpierre 1998 / Cavailhès et al. 2004

A city like a Sierpinski carpet is more interesting than a compact city for people who visit frequently small and medium size centers, but not so often the main center, and who want to be close to open spaces.

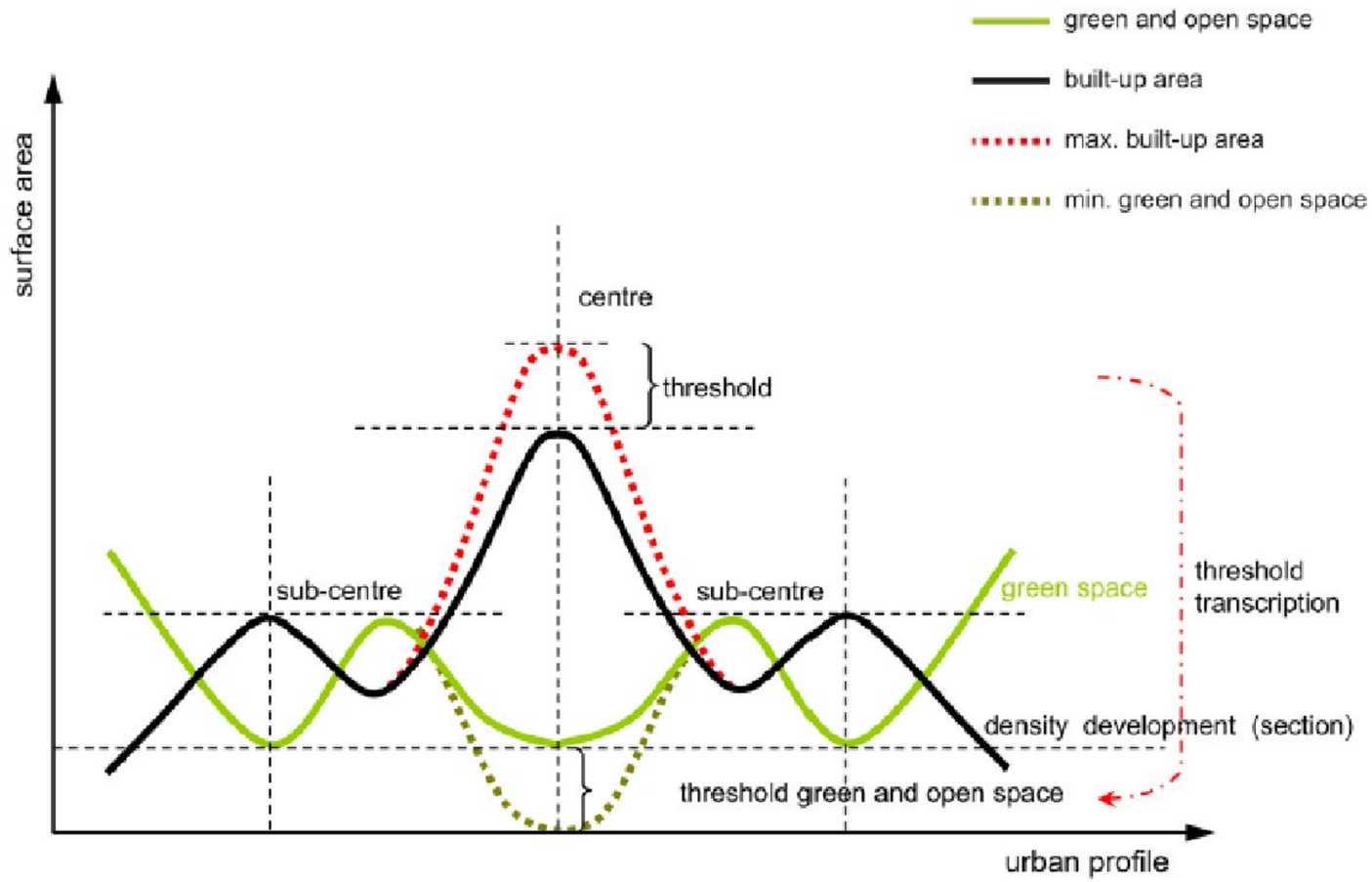
Multifractal logic

Combining several factors of reduction

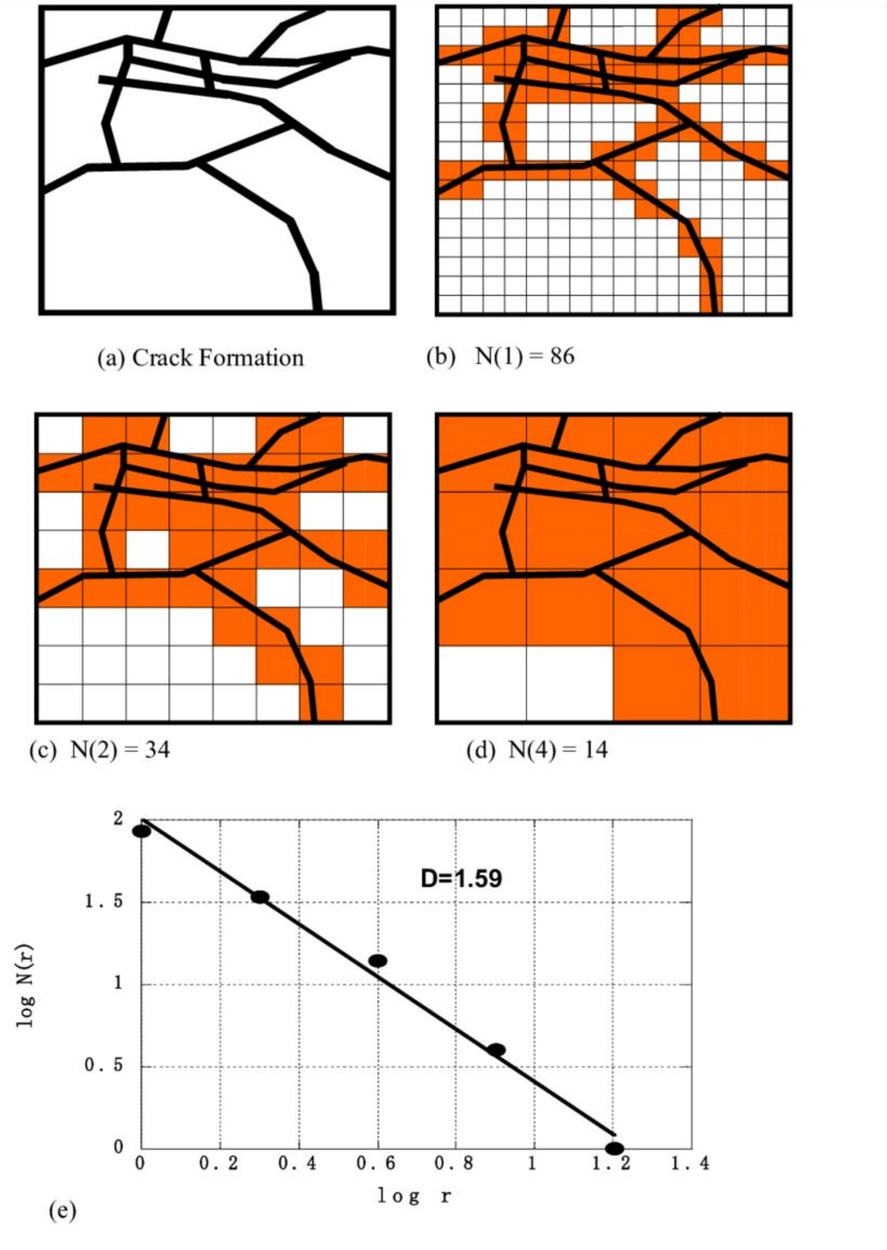


- generates a hierarchy of elements and lacunas
- iteration mixes the reduction factors
- however degeneration since $r_1 r_0 = r_0 r_1$
- Described by a series of fractal dimension $D_0, D_1, D_2, D_3 \ldots$

The multifractal urban profile



Box counting method in Architecture



Adavantage

- It is simple and manually computable

Challenge

- Edge effect on the grid
- Scaling box size (linear versus expotential)
- Grid positioning and orientation

Multi-scale, multifractal urban planning models

– Hierarchical (polycentric) urban development to manage urban sprawl

– Sustainable transit-oriented development

– Locally well-balanced urban patterns and functions

– Marriage of green areas and built-up areas

– Preservation of large interconnected networks of green areas to conserve biodiversity

– Urban (micro)climate

Operationalisation for planning: Fractalopolis

A BRIEF HISTORY

- First theoretical considerations in 2008 (Frankhauser)
- Operationalisation and software development from 2008-2015 (van der Laag Yamu, Frankhauser, Vuidel)
- Calibration of the model from 2016-2021
- Plugin for QGIS from 2021-2023 (Frankhauser, van der Laag Yamu, Bonin, Vuidel, Regrain)

Others

B Planning and Design

Environment and Planning B:
Planning and Design
2016, Vol. 43(6) 969

DOI: 10.1177/0265813516672551
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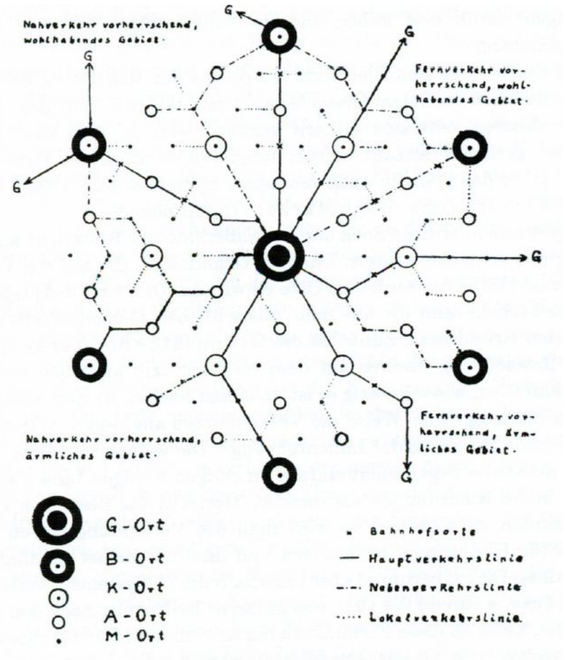
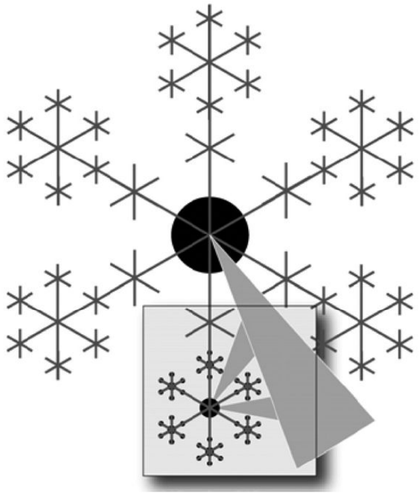
Winners of the Breheny Prize

The editors of *Environment and Planning B* would like to announce that the Michael Breheny prize for the best paper published in 2015 has been awarded to Claudi Yamu and Pierre Frankhauser for their paper 'Spatial accessibility to amenities, natural areas and urban green spaces: using a multiscale, multifractal simulation model for managing urban sprawl' which was published in the journal in November 2015, vol. 42, no. 6, 1054–1078.

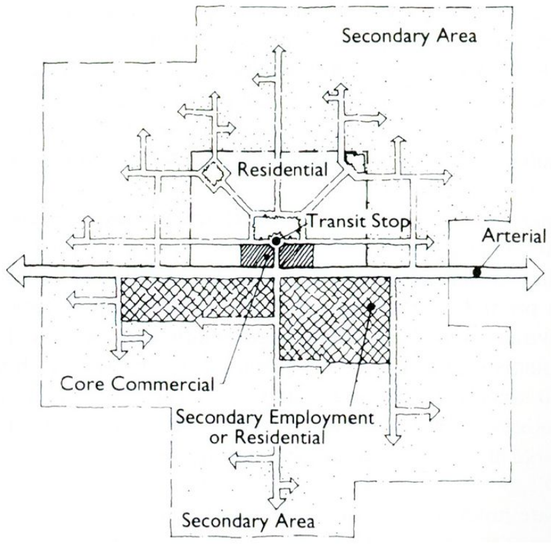
Multifractal Christaller



Christaller`s transport network (1933)

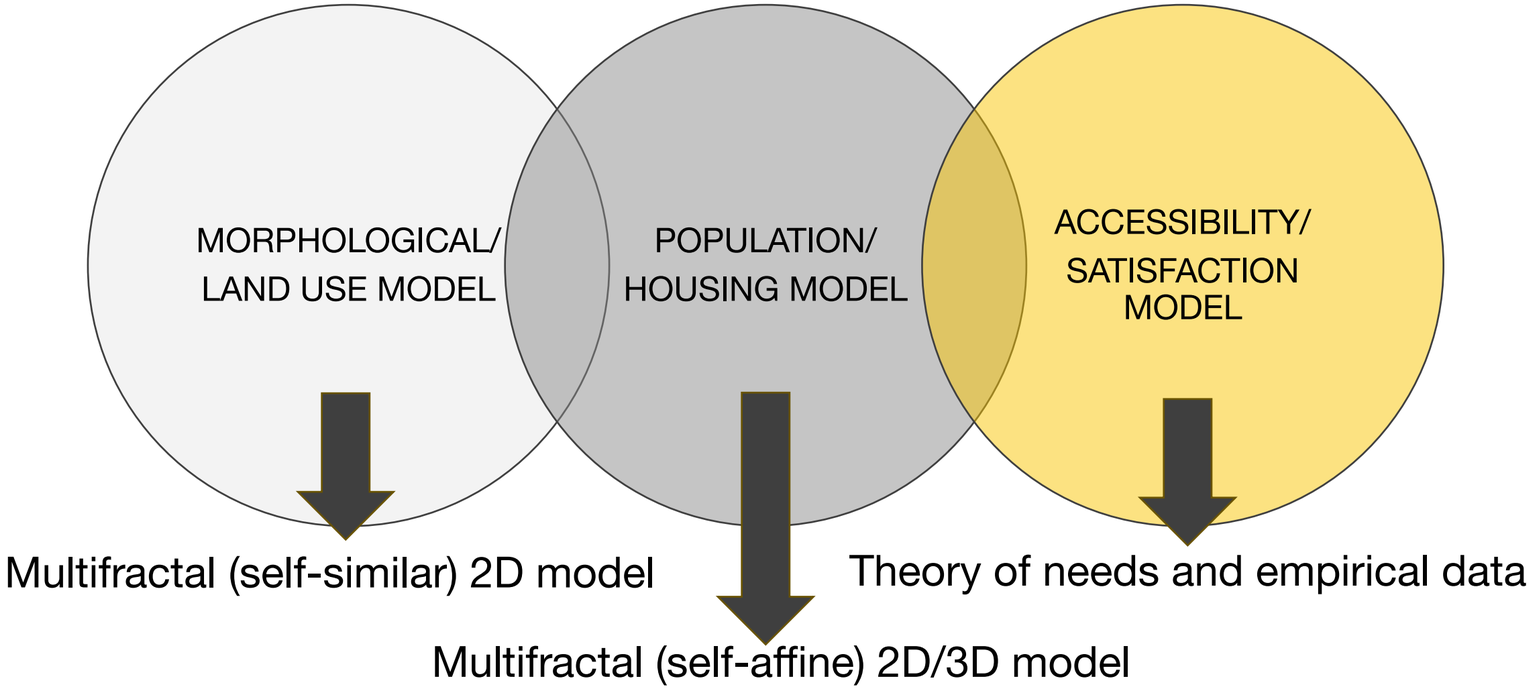


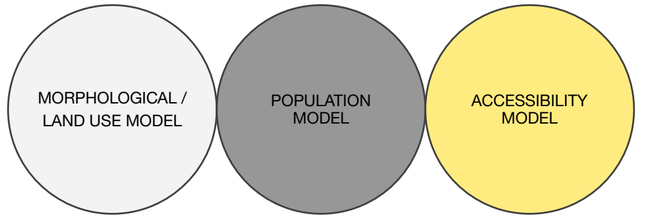
Frankhauser`s multifractal interpretation of Christaller (2007)



Calthorpe`s local TOD model (1993)

The three pillars of Fractalopolis





Morphology

Accessibility

Land use

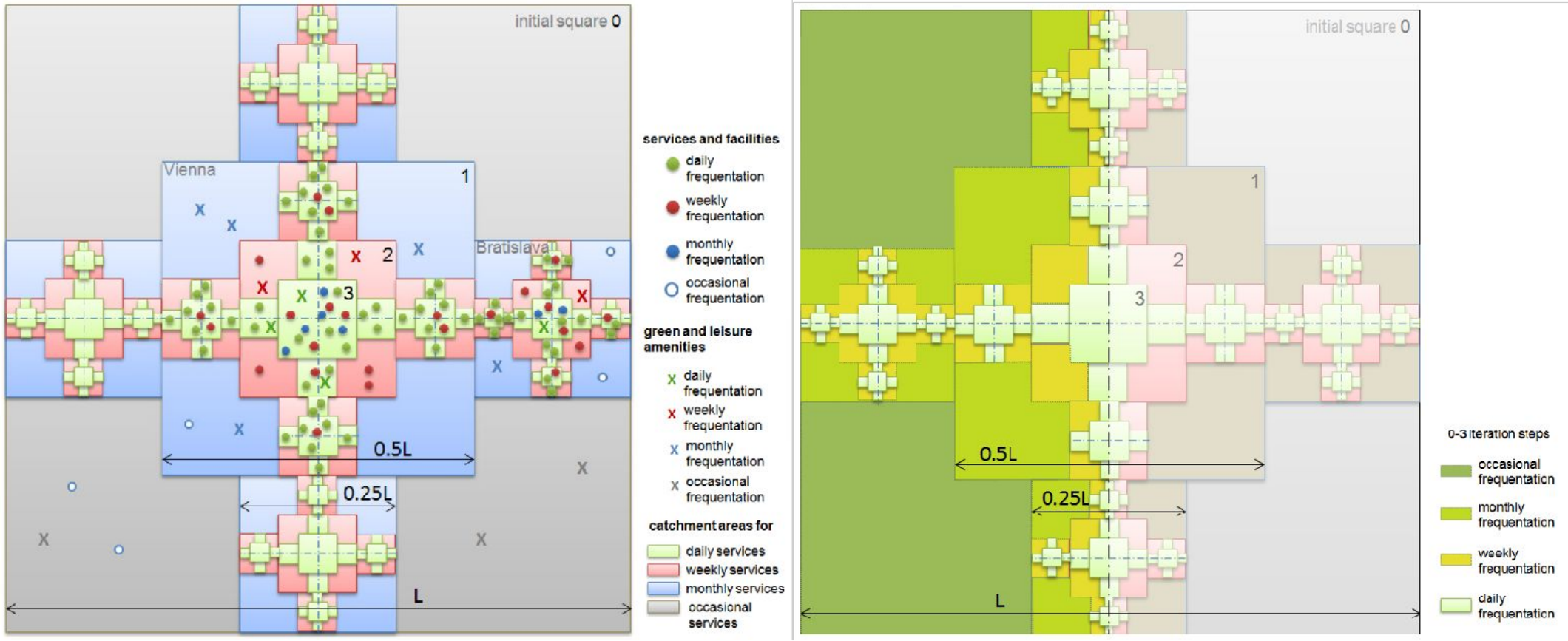
Green areas

Density

Challenge	Rule
Morphological standards	
Multiscale intensity of occupation	Fractal iteration rule
No fragmentation of development	Neighbourhood rule
Environmental quality	Lacunarity rule including landscape view
Accessibility standards	
Access to shopping, services, and facilities at different levels	Distance rule (street network); diversity rule, cluster rule
Access to leisure and sports facilities at different levels	Distance rule (street network); diversity rule
Access to natural areas and urban green spaces at different levels	Distance rule (street network); size rule
Development standards	
Central place hierarchy	Distance rule (street network); diversity rule
Preservation standards (natural environment)	
Avoid fragmentation of green areas; protected zones	Neighbourhood rules
Respect hierarchy of green areas	(Multi)Fractal logic
Density standards	
Moderate density according to intensity of occupation	Ponderation rule (population)

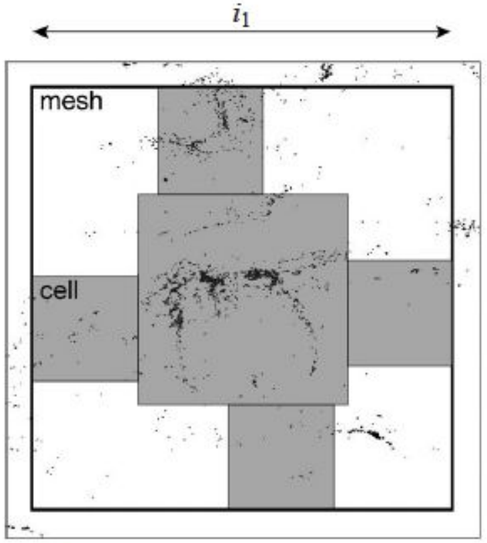
Multi-fractal Christaller

Concept

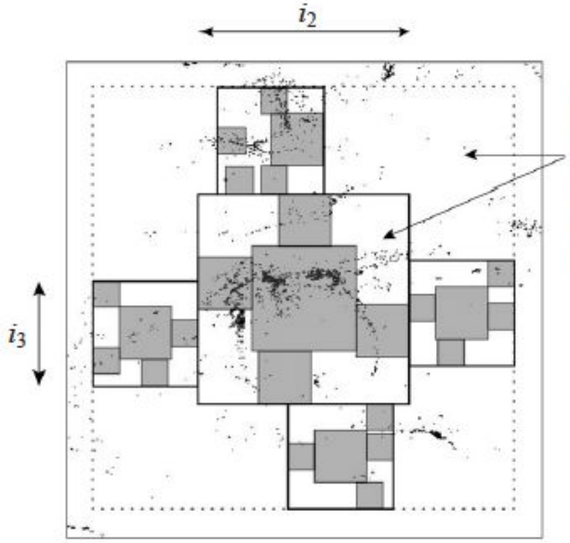


A multifractal spatial model

MORPHOLOGICAL / LAND USE MODEL



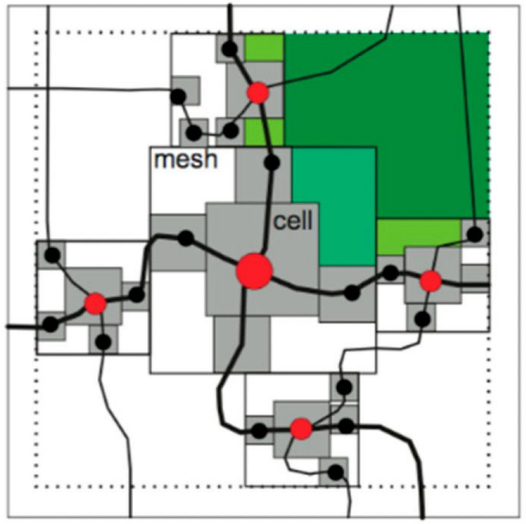
(a)

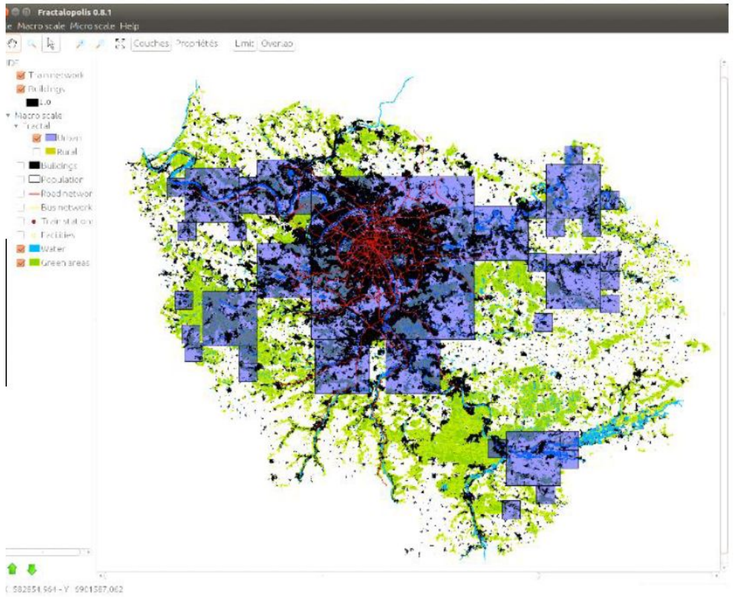


(b)

In this area no urbanization strategy can be carried out as there was no cell located at the previous analysis level and none at the current analysis level ('lacuna')

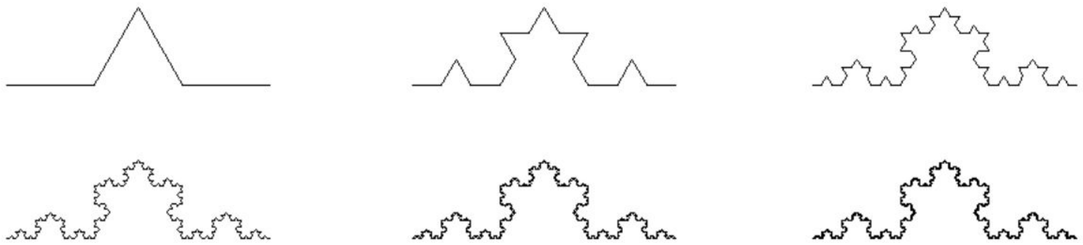
Cells for urbanization
Meshes
Regional level: agglomeration/built-up area
Urban level: quarter
Neighbourhood level: plot



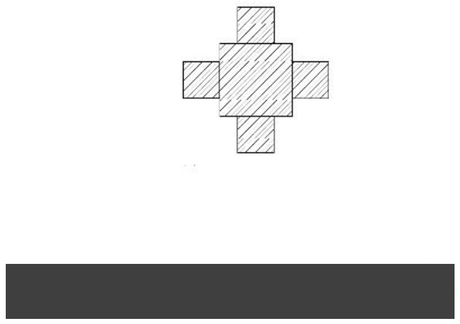


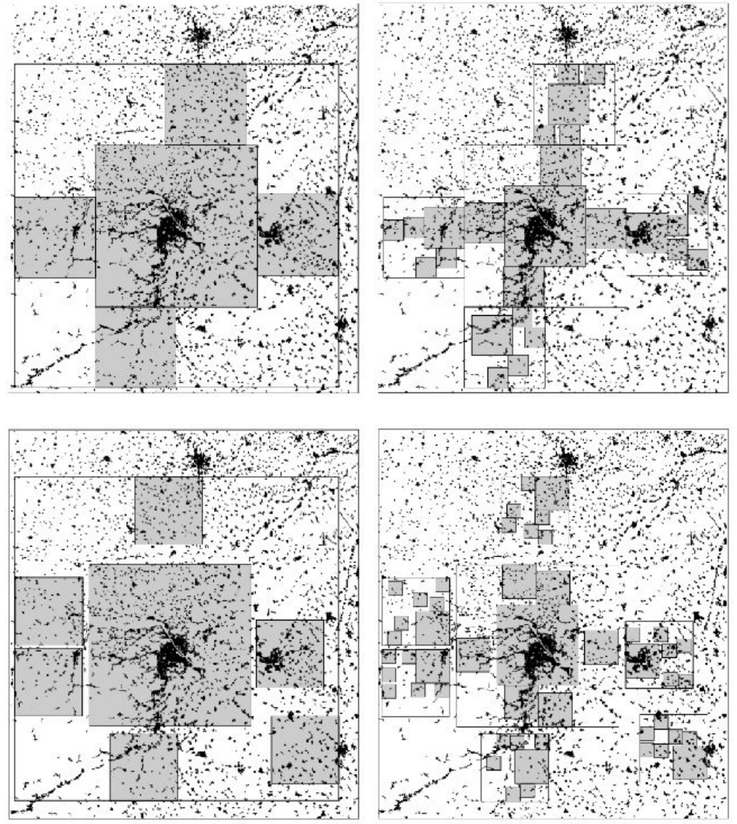
Connecting the spatial scales – Iteration Function System

MORPHOLOGICAL / LAND USE MODEL



Iteration steps of the Koch curve

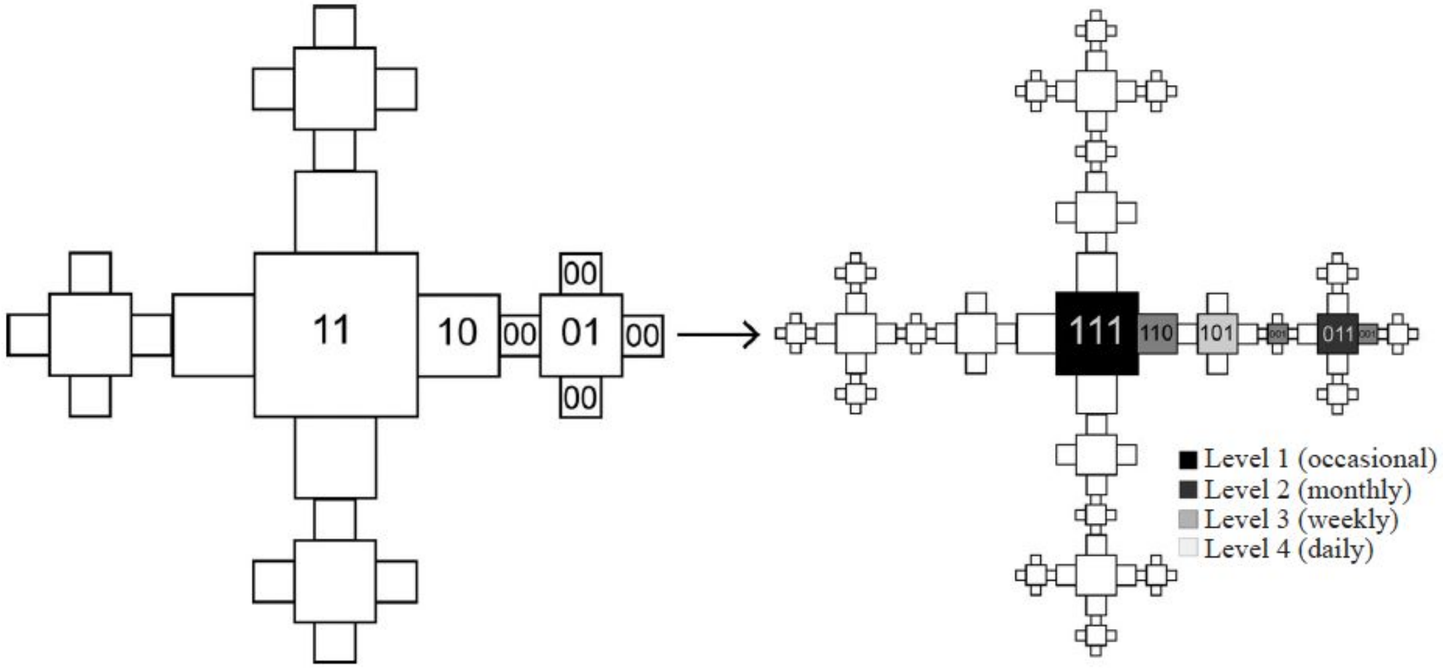




IFS for Fractalopolis

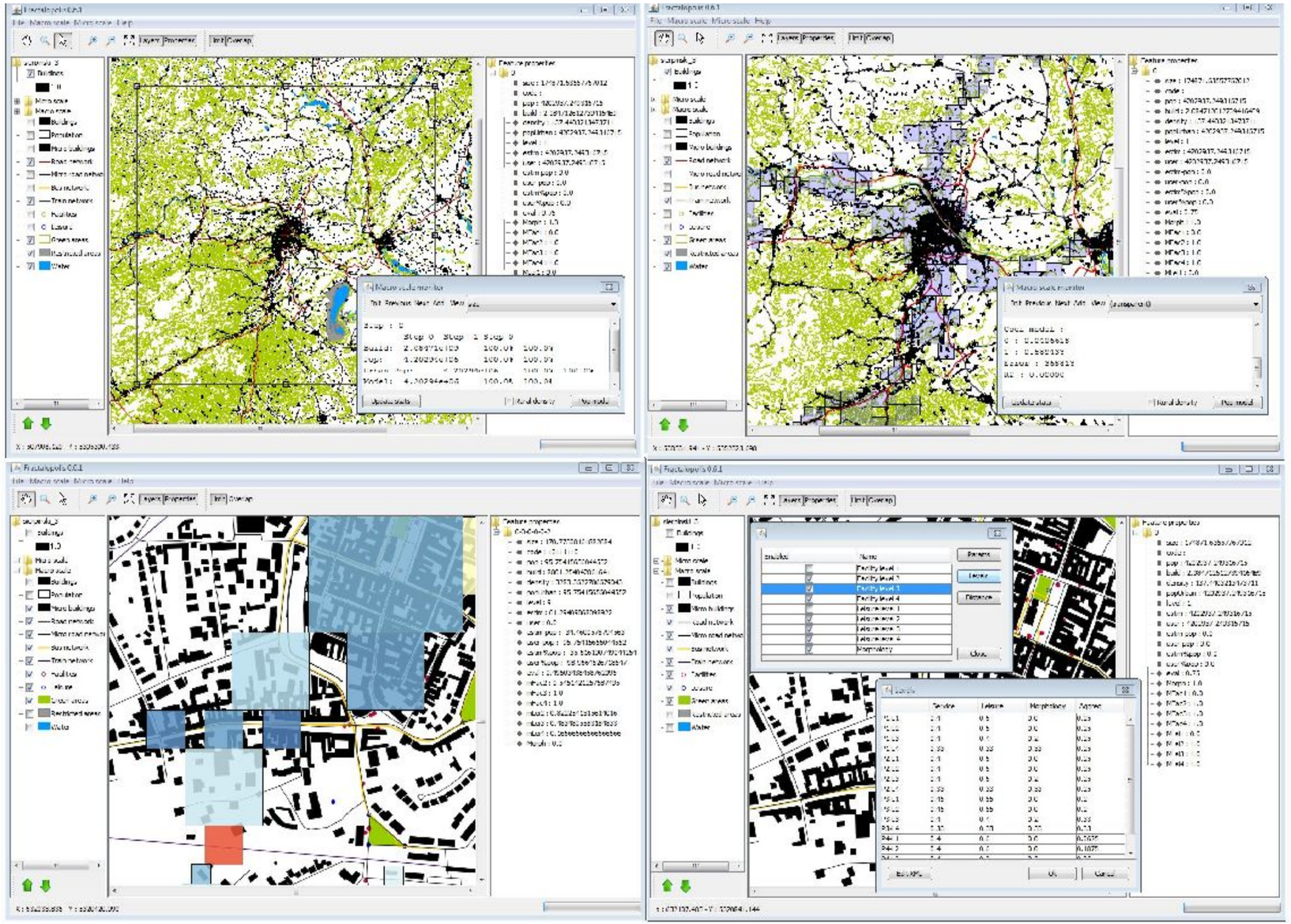
Connecting the spatial scales: The coding system

MORPHOLOGICAL / LAND USE MODEL

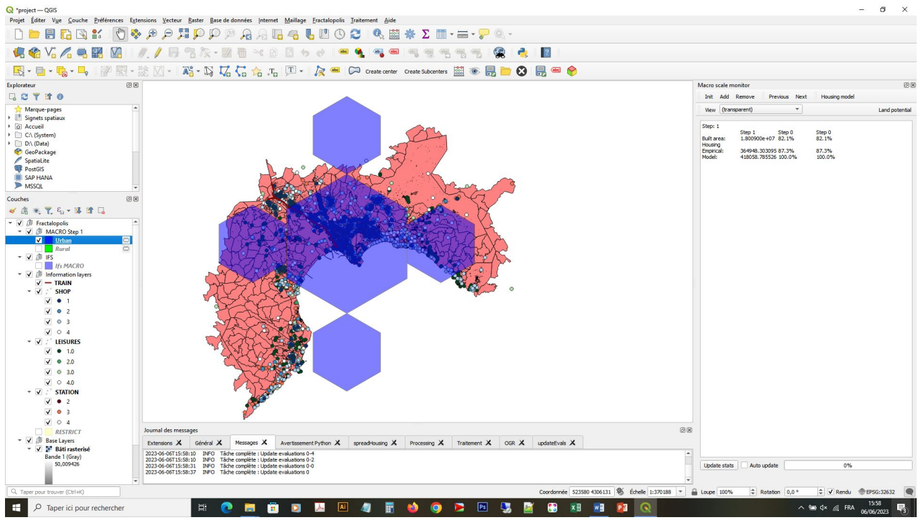


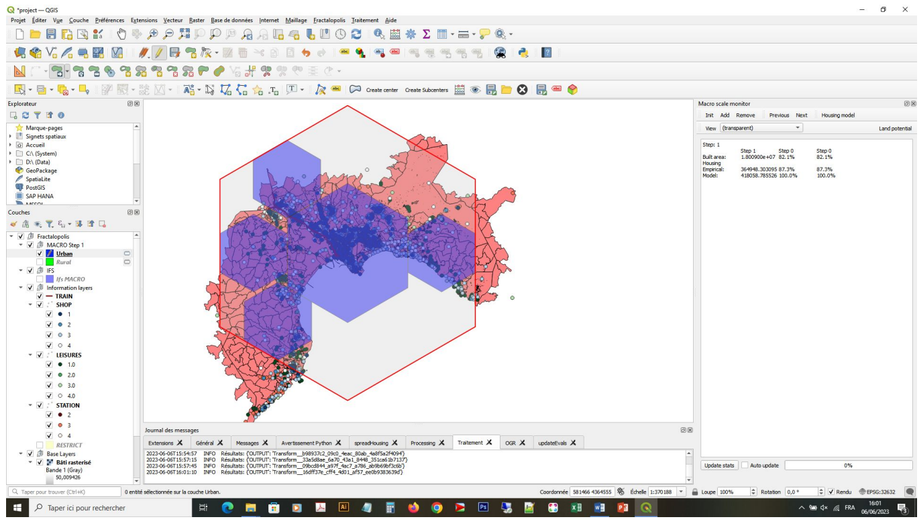
URWISE

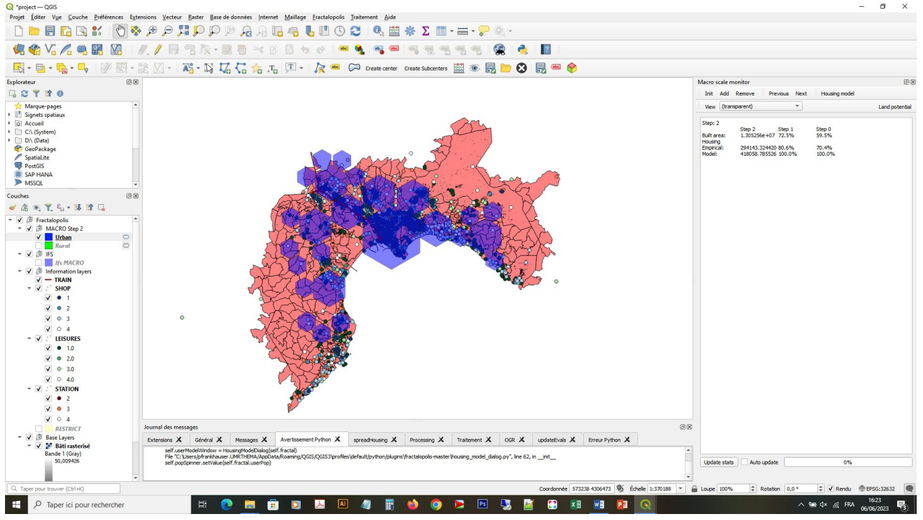
MORPHOLOGICAL / LAND USE MODEL

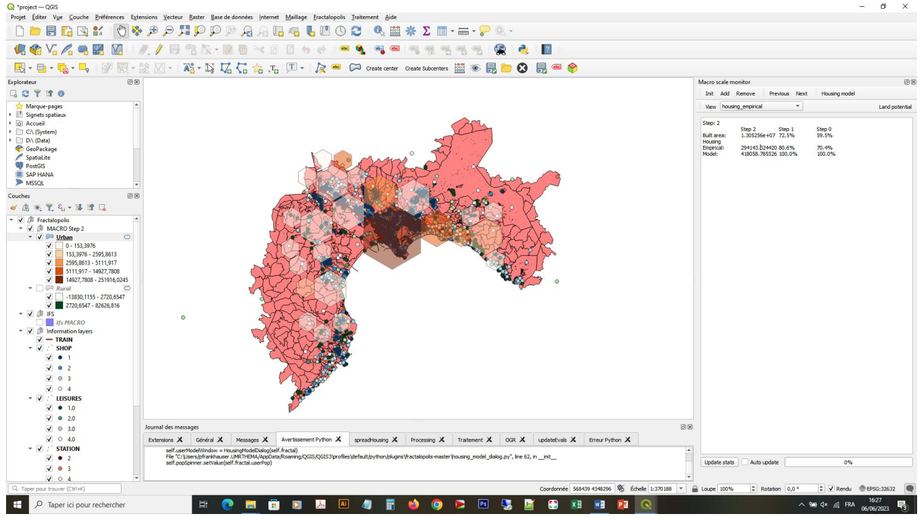


MORPHOLOGICAL / LAND USE MODEL





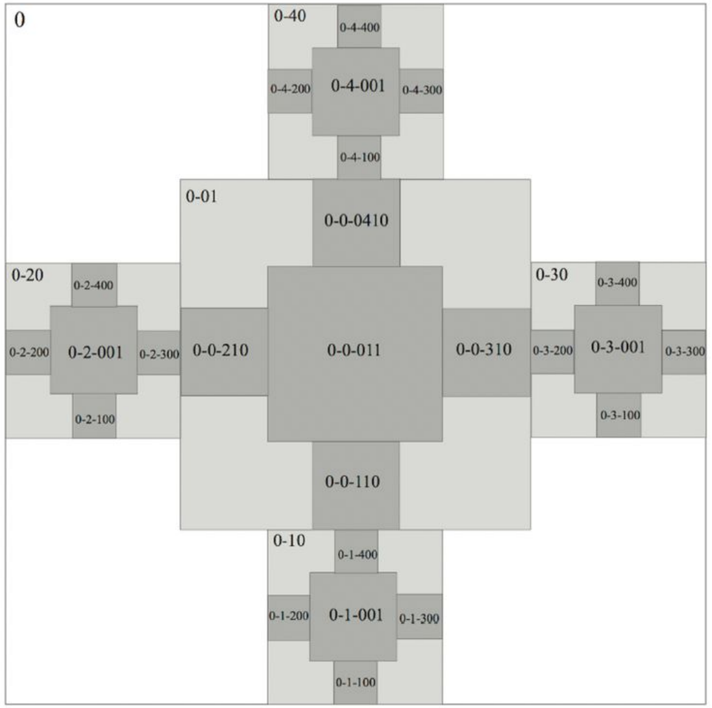




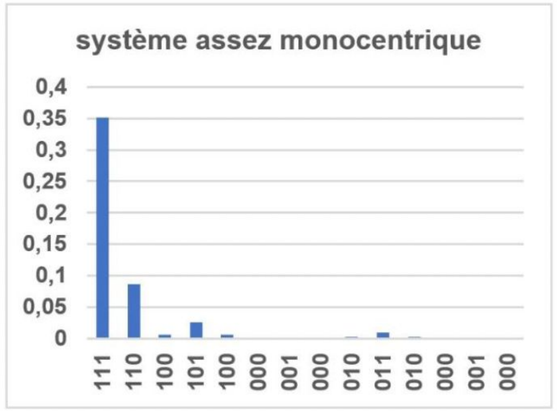
Source: Ongoing project, Garau et al. 2023

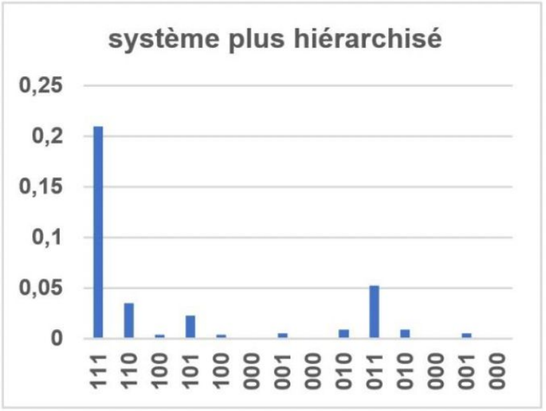
Population model: urban and rural population

POPULATION MODEL

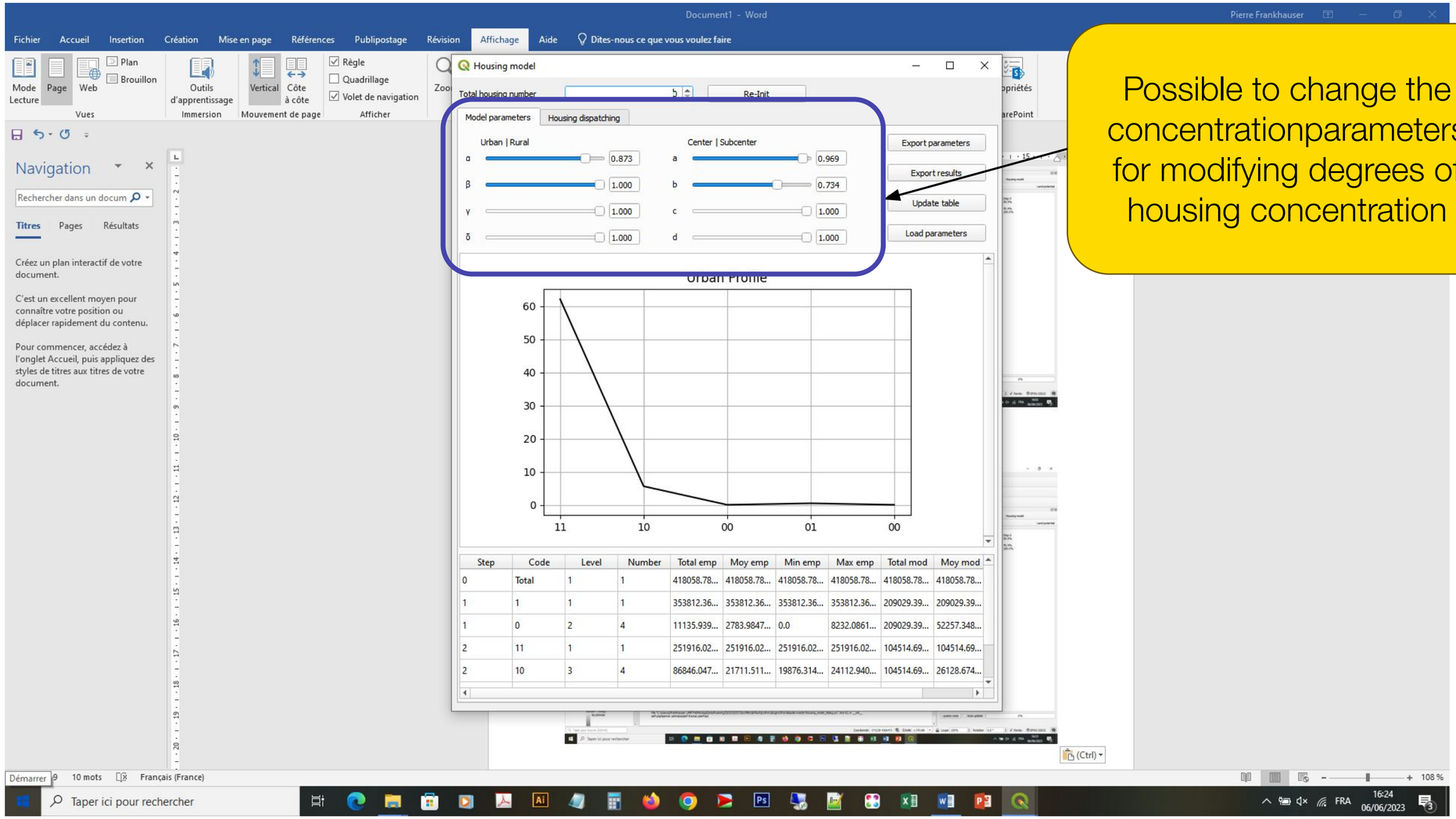


code	level	population	area	density
111	1	$\alpha\beta\gamma p a_1 b_1 c_1$	$(S_1)^3$	$\frac{\alpha\beta\gamma p a_1 b_1 c_1}{(S_1)^3}$
110	4	$\alpha\beta\gamma p a_1 b_1 c_0$	$(S_1)^2 \cdot (S_0)$	$\frac{\alpha\beta\gamma p a_1 b_1 c_0}{(S_1)^2 \cdot (S_0)}$
101	3	$\alpha\beta\gamma p a_1 b_0 c_1$	$(S_1)^2 \cdot (S_0)$	$\frac{\alpha\beta\gamma p a_1 b_0 c_1}{(S_1)^2 \cdot (S_0)}$
100	4	$\alpha\beta\gamma p a_1 b_0 c_0$	$(S_1) \cdot (S_0)^2$	$\frac{\alpha\beta\gamma p a_1 b_0 c_0}{(S_1) \cdot (S_0)^2}$





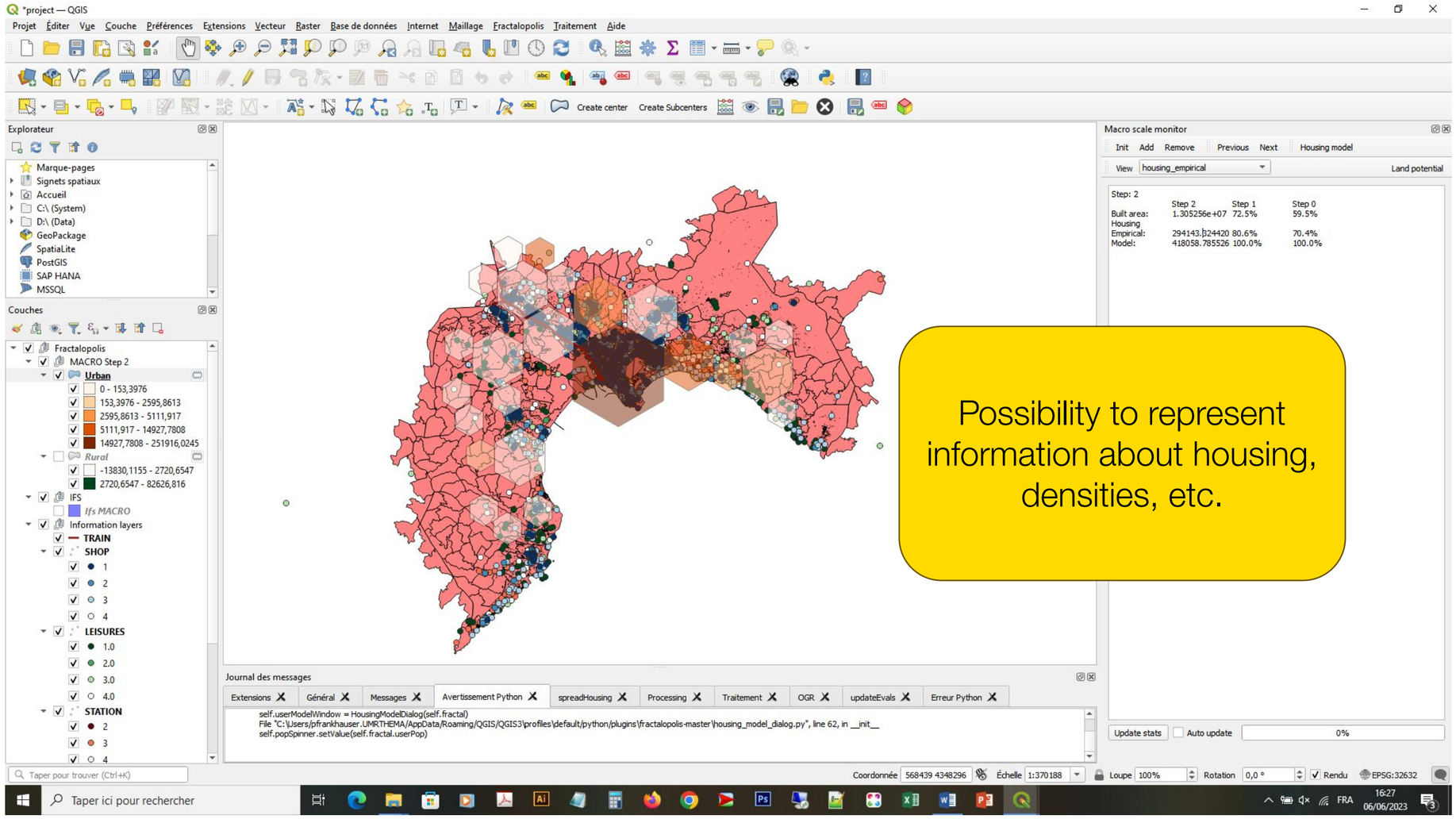
POPULATION MODEL



Possible to change the concentrationparameters for modifying degrees of housing concentration

Population and housing model: urban and rural population

POPULATION MODEL



Possibility to represent information about housing, densities, etc.

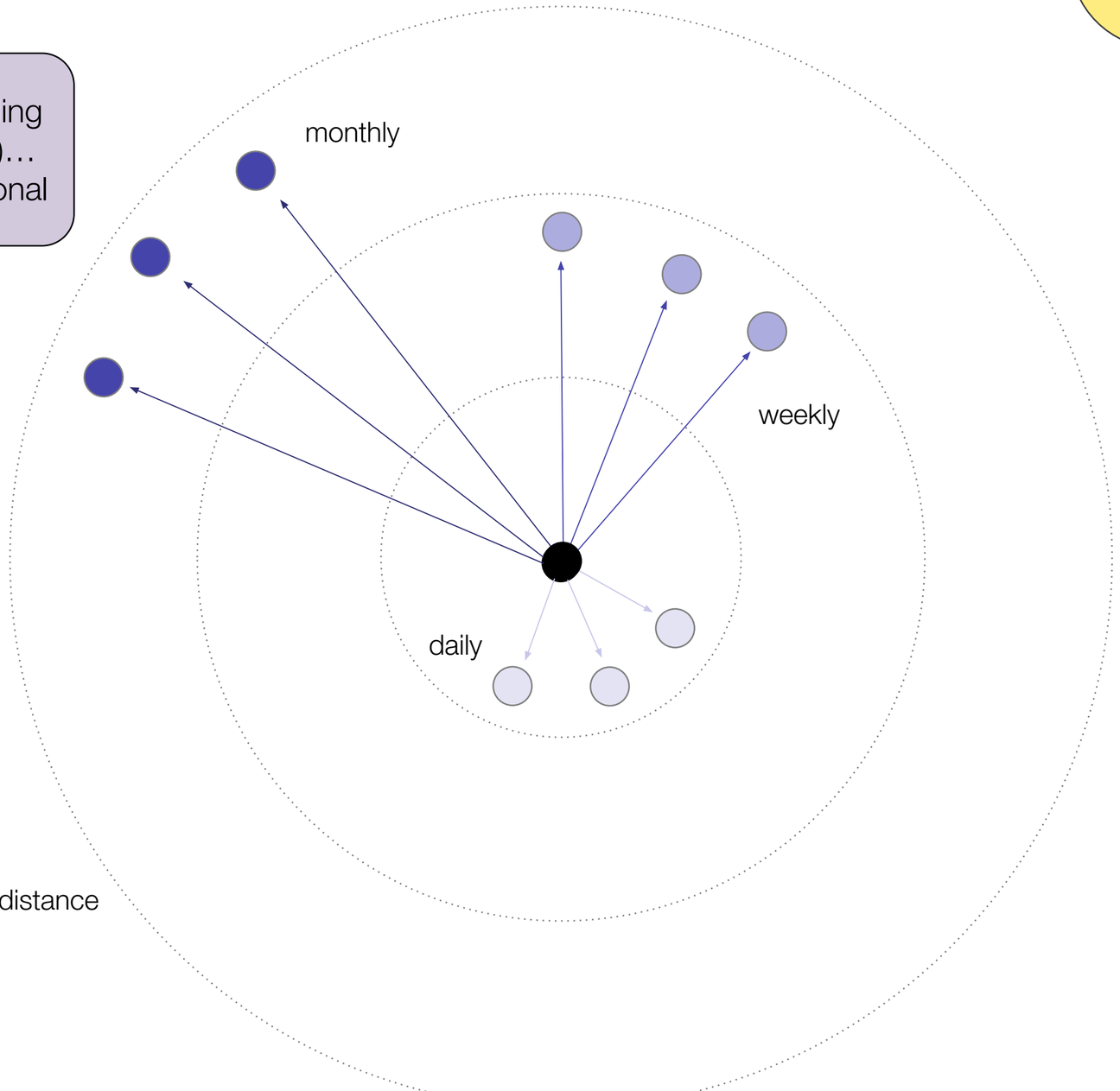
Satisfaction model: Accessibility and catchment area

ACCESSIBILITY MODEL

Accessibility of services acoording to the ranking of needs (shops)… daily, weekly, monthly, occassional

Evaluation of profitability

Formal concept of people`s satisfaction for a given area



Accessibility model

ACCESSIBILITY MODEL

Hierarchy of needs *Frequency of attendance*

Satisfier *amenity*

need

Empirical data about acceptance of distances

Empirical data about percentage of residents using different kinds of amenities

2	occasionnelle	Médecin spécialiste	Protection	3000-6000	48,[illegible]%	10
		Kinésithérapeute	Protection		19%	10
		Psychologue	Protection		6%	10
		Dentiste	Protection		54,6%	10
		Cardiologie	Protection		9,1%	13
		Dermatologie	Protection		10,6%	13
		Ophtalmologie	Protection		26,4%	13
		Gynécologie-obstétrique	Protection		15,4%	13
		Oto-rhino-laryngologie (ORL)	Protection		6,8%	13
		Psychiatrie	Protection		2,4%	13
		Pédiatre	Protection		17,8%	13
		EHPAD	Protection		0,9%	11
		Services municipaux (Mairie/Centre d'Action Sociale (CCAS, CIAS)	Protection		39%	1
		Hypermarchés	Subsistance		84,9%	2
		Marchés	Subsistance		67,9%	2
		Jardinerie	Subsistance		29,4%	6
		Instituts de beauté	Oisiveté		27%	5
		magasin spécialisé habillement	Subsistance		88%	1
		Bibliothèque	Compréhension, oisiveté		27%	12
		Cinéma	Oisiveté, compréhension		63%	12

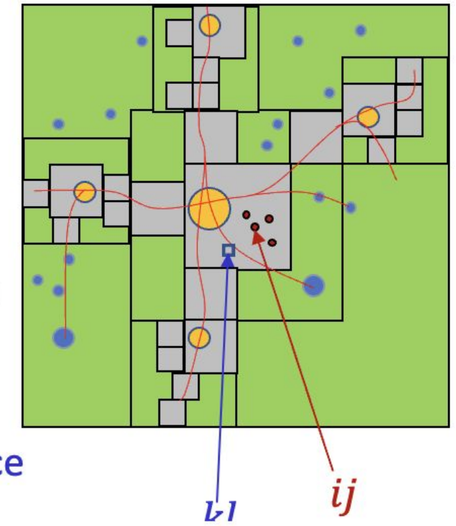
Accessibility model

ACCESSIBILITY MODEL

- Compute for each Fractalopolis zone the level of potential satisfaction of residents
- Satisfaction computed for each hierarchical level, then agregated

Procedure:

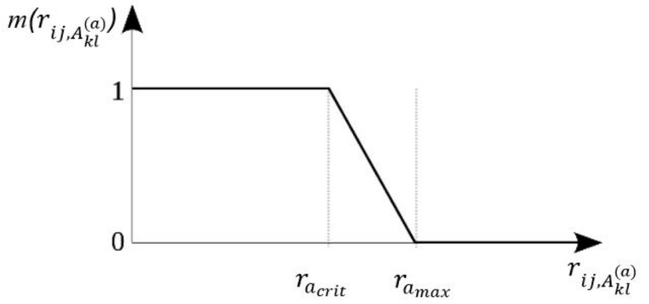
- Selecting the planning area
- Consider amenities $A_{kl}^{(a)}$ of type *(a)* localized in $x=k$, $y=l$
- For each dwelling l_{ij} localized in $x=i$, $y=j$ within the area compute the potential satisfaction level $m(l_{ij}, A_{kl}^{(a)})$:



$$m\left(l_{ij}, A_{kl}^{(a)}\right) = m(a)\, m(r_{ij,A_{kl}^{(a)}})$$

distance acceptance

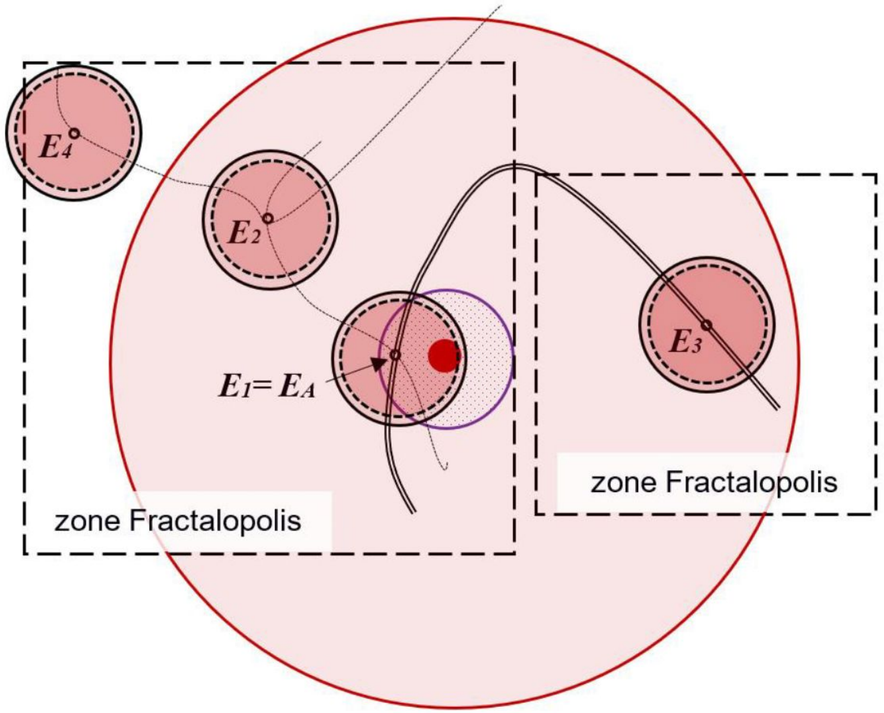
ratio of residents using different kinds of amenities

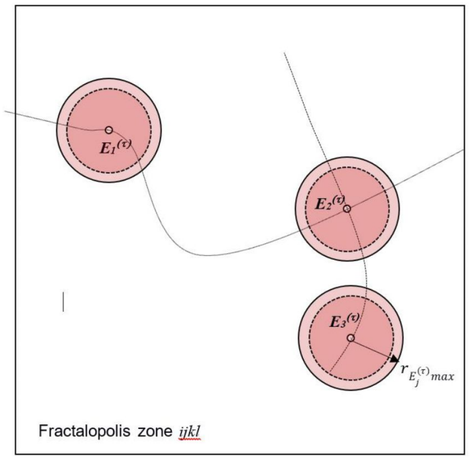


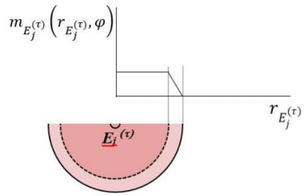
Source: Pierre Frankhauser 2023

Accessibility model

ACCESSIBILITY MODEL





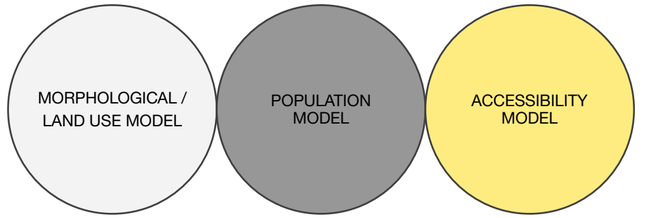


$$m_{zone\ ijkl}{}^{(4)}(\tau) = \frac{1}{s_{zone}} \iint_{zone\ ijkl} \frac{1}{\tau_{max}} \sum_{\tau=1}^{\tau_{max}} m\left(E_j^{(\tau)}, \boldsymbol{r}\right) d\boldsymbol{r}$$

$$m\left(E_j^{(\tau)}\right) = m_n(\tau) \iint_{\varphi, r_{E_j^{(\tau)}} \in zone\ ijkl} \rho_A\left(r_{E_j^{(\tau)}}, \varphi\right)\ m_{E_j^{(\tau)}}\left(r_{E_j^{(\tau)}}, \varphi\right) r_{E_j^{(\tau)}}\, dr_{E_j^{(\tau)}}\, d\varphi$$

Source: Pierre Frankhauser 2022

Synthetic value for an integrated planning approach



Level P_4

$$A(P_4)[F_4] = \mu(S_4) \cap \mu(L_4) \cap \mu(M_4) = 0.33\mu(S_4) + 0.33\mu(L_4) + 0.33\mu(M_4)$$
$$A(P_4)[F_3] = \mu(S_3) \cap \mu(L_3) \cap \mu(M_3) = 0.4\mu(S_3) + 0.3\mu(L_3) + 0.3\mu(M_3)$$
$$A(P_4)[F_2] = \mu(S_2) \cap \mu(L_2) = 0.4\mu(S_2) + 0.6\mu(L_2)$$
$$A(P_4)[F_1] = \mu(S_1) \cap \mu(L_1) = 0.4\mu(S_1) + 0.6\mu(L_1)$$

Accessibility to functions – temporal and weighted include. morphological model

Christaller

Oberzentrum P_1 *to amenities* F_4, F_3, F_2, F_1:

$$A(P_1) = 0.25A(P_1)[F_4] + 0.25A(P_1)[F_3] + 0.25A(P_1)[F_2] + 0.25A(P_1)[F_1]$$

Mittelzentrum P_2 *to amenities* F_4, F_3, F_2, F_1:

$$A(P_2) = 0.25A(P_2)[F_4] + 0.25A(P_2)[F_3] + 0.25A(P_2)[F_2] + 0.25A(P_2)[F_1]$$

Unterzentrum P_3 *to amenities* F_4, F_3, F_2, F_1:

$$A(P_3) = 0.3A(P_3)[F_4] + 0.3A(P_3)[F_3] + 0.2A(P_3)[F_2] + 0.2A(P_3)[F_1]$$

Kleinzentrum P_4 *to amenities* F_4, F_3, F_2, F_1:

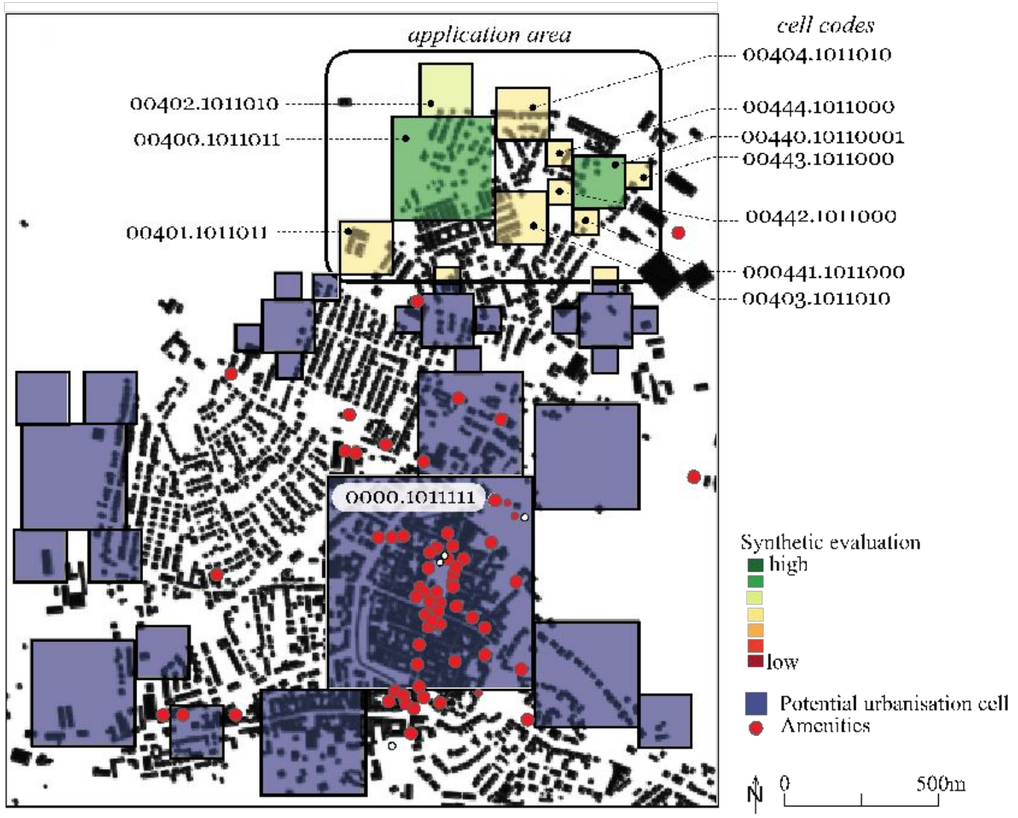
$$A(P_4) = 0.5A(P_4)[F_4] + 0.25A(P_4)[F_3] + 0.1875A(P_4)[F_2] + 0.0625A(P_4)[F_1]$$

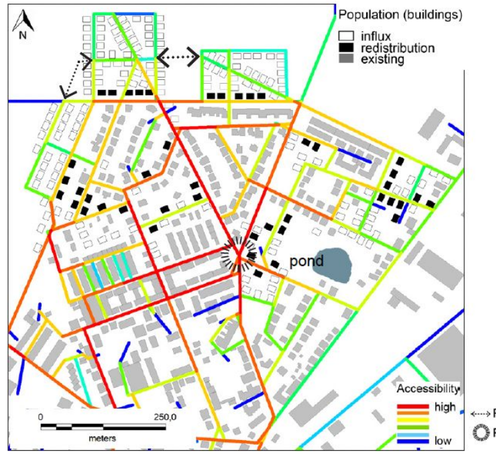
Synthetic evaluation

MORPHOLOGICAL / LAND USE MODEL

POPULATION MODEL

ACCESSIBILITY MODEL

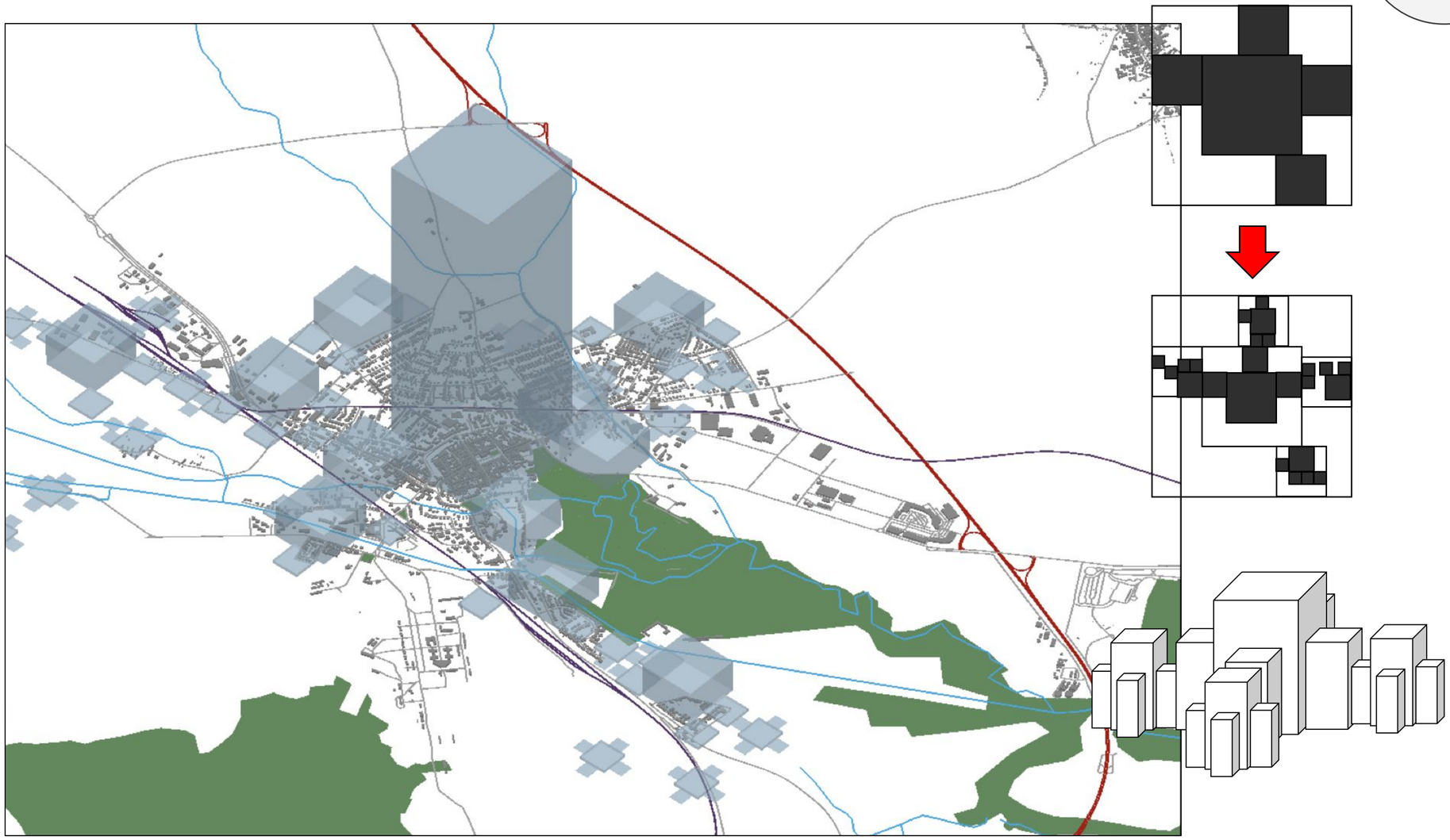




3D Visualisation

MORPHOLOGICAL / LAND USE MODEL

POPULATION MODEL



Why fractals are not mainstream in urban planning

…fractal analytical techniques have been a constant in mathematical urbanism and became popular for innovative methods and techniques, but still does not belong to mainstream approaches herein.

The reason is simply explained: To apply fractal analytical techniques in architecture and urban planning asks for an interpretation of the fractal dimension transferred to architectural design and urban scenarios rooted in complexity science. The logic that the fractal dimension is explaining spatial change from one spatial level to another spatial level via a "surface occupation" index (which is not a generalised/overall density as defined in urbanism) needs a holistic understanding of physics, mathematics and is often cumbersome in its interpretation for the spatial and even more socio-spatial logic for the built environment.

Is the fractal approach useful to create future cities and regions?

Thank You!

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